Hotelling's competition with demand location uncertainty

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Abstract

Firms' product variety choices are examined in a differentiated products model where the range of consumers' preferences is an unknown (strict) subset of the range of potential innovations. It is found that the firms, locating sequentially, will follow the basic pattern of locations found by Neven (International Journal of Industrial Organization, 1987, 5, 419-434). The uncertainty, however, generally causes a greater, more symmetric, amount of differentiation among varieties as firms gamble on finding a market niche with less fierce competition. For a sufficiently large pool of potential entrants, some firms will lose this gamble and will fail.

Keywords: Hotelling's line; Uncertainty; Product differentiation

JEL classification: L10; O30

1. Introduction

History is full of examples of innovations that, despite high hopes, never seemed to catch much of the buying public's attention. Still, ever since the seminal article by Hotelling (1929), the differentiated products literature has generally assumed that potential innovators could be certain of facing some demand for any given innovation which might occur. The typical differentiated products paper allows entrants to choose product varieties given by locations along a line segment that is populated by consumers who are distributed uniformly over the entire interval. This paper assumes that the consumers' preferences do not cover the entire space of possible innovations, but rather an unknown (strict) subset of that range.
In a recent article, Neven (1987) simulated a sequential entry game in a Hotelling's line model. There, the consumers were spread uniformly over the entire interval of possible innovations – the unit interval. The purpose of this work is to extend that paper by comparing its results with those occurring in a simulation when the interval of varieties that consumers most prefer is not known with certainty. Instead, the firms know only the distribution from which the location of the uniform interval of consumers' tastes will be drawn. While it might be expected that the firms would become more likely to bunch in the center of possible innovations where there is a higher probability of being able to sell the chosen innovations, the firms here will generally be found to locate almost evenly throughout the interval of possible innovations, as later firms risk not selling to any consumer in order to decrease the intensity of the competition if they are able to sell.

A firm has an incentive to locate its product variety as far as possible from its competitors' product varieties in order to decrease price competition. The addition of uncertainty, however, weakens that incentive since increasing the differentiation often means increasing the probability that no consumer would consume the variety. If the firm tries to differentiate its product greatly from others, it might have a low probability of earning positive profits, but those profits will be high if it does. In general, a firm is willing to accept some lower probability of facing positive demand when it ensures that some other firm will not be a competitor.

This paper is similar to one by Jovanovic (1981). That paper also looked at demand location uncertainty, but with some differences. First, each firm had private information on the consumers' locations consistent with a single observation of a consumer's most-preferred good. This implies, in part, that every producing firm produces a variety that some consumer prefers to all others. Thus, no firm finds itself in the situation of innovating, but then failing to make a profit. In the present paper, the uncertainty about the location of demand without private information means that some innovating firms can and often do fail to sell their products. Second, there was no price competition in Jovanovic's paper. Price competition is one of the main engines pushing for increased product differentiation here. Third, Jovanovic's model was one of simultaneous entry. Neven's work makes use of the sequential entry developed from Hay (1976) and Prescott and Visscher (1977), as will this paper.

The rest of this paper is organized as follows: Section 2 gives a brief description of the model and discusses some of the assumptions; the results of the simulation are given in Section 3; and Section 4 concludes.

\[1\] In discussing the robustness of his results, Jovanovic did mention the extension that the observation could lie outside the range of consumer preferences so that a firm's probability of positive profit would fall below one (p. 657).
2. The model

The consumers are spread uniformly over some interval, \([\Theta, \Theta + 1]\). The utility of the consumers is measured in dollars, and quadratic transportation costs are used (e.g. a consumer located at \(a\) who buys from a producer located at \(x\) will lose utility of \((x - a)^2\) in addition to the price paid). In addition, utility is specified so that each consumer buys, at most, one unit from this class of goods; and the reservation price, \(s\), for each consumer is sufficiently high (\(s > 3\), here) that consumers will always buy at least one unit.

There is an exogenously determined number of competing firms, \(N\). These firms do not know the value of \(\Theta\) a priori, but only that \(\Theta\) is drawn from a uniform distribution, \([0, 1]\). If a firm chooses a product variety that is outside the interval \([\Theta, \Theta + 1]\), then that firm's profits are assumed to be zero. Thus, firms know a priori that any variety location outside the interval \([0, 2]\) is guaranteed to earn zero profits.

The firms enter the market sequentially, choosing single locations \((x_1, x_2, x_3, \ldots, x_N)\) on the real number line (again, they will restrict themselves to the interval \([0, 2]\) in order to have the possibility of earning profits). After all have entered, the firms engage in price competition. The equilibrium is, thus, subgame perfect since all firms are aware of the price equilibrium that would result from any configuration of varieties and \(\Theta\). The equilibrium is restricted to pure strategies.

For simplicity, all costs are assumed to be zero.

In this model, all firms choose varieties before the price competition begins. A firm announces its variety before production begins. Other firms react to the potential profits by also announcing their varieties before any sales are made. For reasons of reputation or a high cost of reconfiguring product development, the firms find it prohibitively expensive to change product varieties once announced. Later firms know this and assume the earlier firms will produce their announced varieties. In computer software development, as just one example, firms often announce the coming of new programs well in advance of their actually hitting the market. This allows other firms to respond before demand has been fully determined.

The assumption that any firm location outside the interval \([\Theta, \Theta + 1]\) earns zero profits is borrowed from Jovanovic. While it is easy to think of examples where such a firm would not earn zero profits (a monopolist at \(\Theta - \epsilon\), for instance), the assumption is made in order to be able to apply the proof of existence and uniqueness of the pricing equilibrium found in Neven's paper. This assumption is not unreasonable as the number of firms becomes large, since the profits of firms producing goods that are not preferred by any consumers will go to zero. Also, since the usual paper in this literature (see e.g. d'Aspremont et al., 1979) does not allow firms to enter where no consumers are located, the effect is the same as this assumption — no firm profitably enters outside the interval of consumers.
In Lane (1980) – from which Neven draws – the existence of equilibrium in the complete location pricing game cannot be proven, and so a simulation is done. Neven, too, offers a simulation. In order to be able to compare the results with those of Neven, this paper will also employ a simulation.

3. Results

In Neven's paper, the early entrants locate at or near the center of the space of possible innovations. Later entrants differentiate their products, locating closer to the boundaries of the interval. Firms locate so that all subsequent entry – all on that half of the interval – will be toward the boundary. In this simulation, that pattern again results. There is, however, a stronger incentive to locate away from others' locations, leading to greater differentiation of varieties (except in the duopoly case).

The firms here have conflicting incentives. There is an incentive to locate near the center of the interval where the probability of being inside the interval \([\Theta, \Theta + 1]\) is higher. There is, however, a pull to the outside – away from the competitors – in order to decrease the price competition and to decrease the number of firms in the market if \(\Theta\) is favorable. In other words, moving away from the competition will probably make it less likely that the firm will be able to make profits at all, but if it is able to make profits, those profits will be higher than they would be otherwise. In general, the pull away from the other firms is more powerful with the uncertainty of demand since it is possible that the firm has correctly guessed the realization of \(\Theta\) while others did not.

Table 1 gives equilibrium varieties with exogenous sequential entry.

### 3.1. Monopoly

For a firm which knows it will be a monopoly, the center of the \([0,2]\) range is clearly the optimal location. Not only is that the expected center of the consumers,

<table>
<thead>
<tr>
<th>Neven (1987)</th>
<th>Harter</th>
<th>(E\pi_1)</th>
</tr>
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<tbody>
<tr>
<td>(x_1 = 0.5)</td>
<td>(x_1 = 1.00)</td>
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<tr>
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<td>(x_1 = 1.01)</td>
<td>2.31758</td>
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<tr>
<td>(x_2 = 0.0)</td>
<td>(x_2 = 0.50)</td>
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</tr>
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<td>(x_1 = 1.00)</td>
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<tr>
<td>(x_2 = 0.891)</td>
<td>(x_2 = 1.50)</td>
<td>0.12745</td>
</tr>
<tr>
<td>(x_3 = 0.068)</td>
<td>(x_3 = 0.49)</td>
<td>0.12741</td>
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<tr>
<td>(x_1 = 0.675)</td>
<td>(x_1 = 1.24)</td>
<td>0.20155</td>
</tr>
<tr>
<td>(x_2 = 0.315)</td>
<td>(x_2 = 0.75)</td>
<td>0.19612</td>
</tr>
<tr>
<td>(x_3 = 0.930)</td>
<td>(x_3 = 1.76)</td>
<td>0.054620</td>
</tr>
<tr>
<td>(x_4 = 0.062)</td>
<td>(x_4 = 0.24)</td>
<td>0.052047</td>
</tr>
</tbody>
</table>
3.2. Duopoly

With duopoly, the first firm locates very near the center, and the second firm locates on the longer side. Since there is a positive probability that the consumers will be everywhere to the one side of the second firm, the first firm has a positive probability of earning monopoly profits. In addition, locating farther away from the second firm (beyond the location of \( x_1 = 1 \)) will decrease the probability that the chosen location will be an element of \([\Theta, \Theta + 1]\), and, thus, of making positive profits for the first firm. The first firm, therefore, does not locate as far as possible from where it expects the second firm to locate as in Neven's work. Thus, unlike the case with certainty of demand location, maximum differentiation does not occur. While it is possible for the realization of \( \Theta \) to be such that only one firm actually sells in the market, the differentiation is less than previously predicted when both firms are able to sell. For example, if \( \Theta = 0 \), then both firms sell in the market, but they will not be maximally differentiated in that market.

Neven's symmetry is a special case resulting "from the discrete interdependence between duopolists" (p. 425). The positive probability of monopoly here keeps the duopoly case from symmetry and maximum differentiation.

3.3. Triopoly

The first firm locates at the center of the interval, and the following firms locate fairly symmetrically around it. However, the differentiation between the firms here is greater than that observed when the location of the consumers is known. This is because the higher profits which could be earned with lower numbers of firms in the market outweigh the loss in the probability of making profits at all. For instance, a firm prefers a 1/2 probability of even the worst possible monopoly to being certain of the best possible duopoly (see Harter, 1993). These preferences continue with larger numbers of firms – a firm prefers having a lower chance of duopoly to a higher chance of triopoly, and so will locate accordingly. Thus, later firms will locate closer to the boundaries in the hopes of finding a higher profit realization of \( \Theta \) rather than locating toward the center where a positive demand is more probable.

Locating closer to other selling firms decreases whatever profits are being made. For instance, the third firm in the three-firm case locates at \( x_3 = 0.49 \) instead of 0.50. If the firm were to locate at 0.50, the probability of being able to sell would go up, but that extra would be a triopoly. The firm would lose some of its duopoly profits due to more intense price competition for \( \Theta \in [0,0.49] \) and would gain relatively small triopoly profits when \( \Theta = 0.50 \). The discontinuity of the expected profits for the third firm at 0.50 causes the third firm to locate just far enough from...
the second firm that it will never have to compete with it for any realization of $\Theta$. The second firm knows this, and locates accordingly. It will not locate too closely to the first firm, or the price competition would become too great. Locating closer to the boundary would decrease the price competition, but at the cost of making positive profits less likely to be achieved. The second firm, then, locates so that the third will locate almost symmetrically to it.

3.4. Four firms

The first firm does not locate at or near the center of possible innovations here. The presence of a fourth entrant dilutes the gains from locating so that the firm is nearly always in the interval $[\Theta, \Theta + 1]$. The second firm is then able to mirror almost the first firm's location. The third and fourth firms' locations, then, will have no direct effect on each other. In all, the firms locate at almost even intervals along the range of possible innovations.

The firms are locating fairly symmetrically in these results. What is more important to notice, however, is that symmetry comes about because later firms are locating so that they are just more than the length of consumer preferences away from a competitor. In other words, the uncertainty of demand is increasing the pull toward the center of possible innovations as firms try to increase the probability of positive sales. As a later firm moves toward the center, however, its expected profits are discontinuous, decreasing at the location where it is exactly the length of the interval of consumer preferences away from another potential entrant. This is because the number of possible competitors increases discretely at that point. At that location, the firm is willing to take the gamble that $\Theta$ is favorable rather than face an increase in the number of competitors. Because the first and second firms are located almost symmetrically, the third and fourth are also located almost symmetrically to each other.

The expected profits are lower for later entrants, so there is still an incentive to early entry. More firms attempting to enter the market, however, do not always lead to lower expected profits for all the firms. The second entrant benefits from having a fourth entrant, because that fourth entrant forces the initial firm away from the center, allowing the second firm to locate so that the probability of making profits is higher. This distinguishes these results from those in Neven's paper. Generally, though, this effect will be dissipated as the number of firms grows larger, since additional entrants will have an increasingly smaller effect on the early entrants.

4. Conclusion

The purpose of this paper is to add some uncertainty about the location of consumer tastes. The simulation of entry shows that the basic pattern follows that
The firms could be expected to differentiate their products more here since the uncertainty of demand allows firms to take chances that consumers' tastes are extreme. Except in the duopoly case, the entrants spread themselves fairly evenly along the range of potential innovations, with later firms locating closer to the edges of what technology allows as they gamble on correctly guessing the consumers' preferences.

It is worth repeating that the early entrants locate at or near the center of the possible profit-making innovations, not necessarily the center of demand. Thus, any firm that locates anywhere except the exact center of the possible profit-making varieties has a strictly positive probability of not being able to sell its product. This, in fact, happens to at least one firm with certainty when even three firms are entering. That such a firm makes zero profit is by assumption, but would be expected to occur with large numbers of entrants (see Section 2). Thus, unlike the previous literature on the Hotelling's line, some firms fail.

The ex post social welfare is generally hurt by the uncertainty. If, say, three firms are attempting to innovate, at most two will be able to sell their products. Thus, the inefficiency due to the transportation costs is greater than it would be if all three knew the location of demand. Even with one firm, that firm locates in the expected center of the \([\Theta, \Theta + 1]\) interval, which might be off center, and thus less efficient ex post than if \(\Theta\) is known. The uncertainty might improve the ex post social welfare in the special duopoly case, however. With two entrants, if the realization of \(\Theta\) is 0.50, then both firms will be producing and the deadweight loss due to the transport costs will be less.

This paper models an industry where firms announce their intended varieties in advance of production. An extension would be to allow later entrants to locate after earlier firms have begun production. This would allow the later firms to observe the pricing decisions of the early entrants. Early entrants would, thus, signal the location of demand, allowing the later entrants to enter with more knowledge of the market. In the current paper, there is no late mover advantage, but there could very well be if the later entrants are more informed.

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\(^2\) Neven's work also looks at the case of endogenous entry and deterrence of entry. While this paper omits that, the effect on those results would likely be similar.
References