Collusion and leadership

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Abstract

The paper explores the role of price or quantity leadership in facilitating collusion. It extends the standard analysis of tacit collusion by allowing firms to make their strategic choices either simultaneously or sequentially. It is shown that price leadership indeed facilitates collusion by making it easier to punish deviations by the leader. In case of pure Bertrand competition, price leadership restores the scope for (perfect) collusion in markets where collusion would not be sustainable otherwise. When firms face asymmetric costs or offer differentiated products, price leadership can also enhance the profitability of collusion—in case of asymmetric costs, the less efficient firm must act as the leader. Finally, such leadership is less effective in case of Cournot competition since, following an aggressive deviation by the leader, the follower would rather limit its own output, making it more difficult to punish the deviation. Still, quantity leadership may enhance collusion when it is already somewhat effective in a simultaneous move setting.

1. Introduction

Price leadership has been documented in many cartel cases. In the Vitamins cartel, for example, the firms designated a company which would lead a price increase. The Decision of the European Commission states:

“The parties normally agreed that one producer should first ‘announce’ the increase, either in a trade journal or in direct communication with major customers. Once the price increase was announced by one cartel member, the others would generally follow suit.”

Other recent industry cases of collusive price leadership include cartels in sorbates, high pressure laminates, rubber chemicals, graphite electrodes, polyester staple and organic peroxides.1 We also conducted a survey of the European Commission’s cartel decisions, and price leadership features in 16 of the 49 cases available online in English as of July 2010 (see Appendix A for a list of these cases and additional details).2

Yet, most of the literature on collusion assumes that firms set their prices or quantities simultaneously, thus excluding the possible emergence of a leader. The purpose of the paper is to explore the role of such leadership, in a framework where firms can move either simultaneously or sequentially.

More precisely, we study collusion in an infinitely repeated setting for different modes of market behavior, namely, Bertrand and Cournot competition. To allow firms to set their strategies in sequence, in each period the competition game includes two stages, in either of which firms can set their prices or quantities. For each type of competition, we contrast the best collusive equilibria obtained in a constrained period the competition game includes two stages, in either of which firms can set their prices or quantities. For each type of competition, we contrast the best collusive equilibria obtained in a constrained game where firms would have to move simultaneously with those attainable when they can move sequentially. We show that leadership indeed facilitates collusion. Intuitively, deviations by the leader can be more immediately punished, which reduces its incentive to deviate and enlarges the set of sustainable collusive outcomes.

In the case of pure Bertrand competition with homogenous products, price leadership allows firms to sustain (perfect) collusion for any value of the discount factor: any deviation by the leader is easily deterred in that case, and it suffices to give a large enough market share to the follower to ensure that it abides as well to the cartel agreement.

1 These 16 cases are decisions which explicitly mention price leadership as part of the implementation of the cartel agreements, and thus provide a conservative estimate. Similar leadership might have prevailed as well in other cases, without being documented in the decisions.

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1 These 16 cases are decisions which explicitly mention price leadership as part of the implementation of the cartel agreements, and thus provide a conservative estimate. Similar leadership might have prevailed as well in other cases, without being documented in the decisions.
The argument extends to asymmetric firms. As Harrington (1991) and Miklos-Thal (2011) show, it is the incentive constraint of the less efficient firm that constrains the scope for collusion. Since price leadership weakens the incentive constraint of the leader, selecting the less efficient firm as a leader allows both firms to achieve greater collusive profits than with simultaneous pricing. The argument also extends to differentiated products, as long as prices remain strategic complements. While the leader may then gain from deviating, its incentive constraint is still relaxed, which allows firms to charge higher prices (although asymmetrically, the leader charging a higher price than the follower). As a result, price leadership can still allow both firms to increase their collusive profits, particularly if they can take turns to be the leader.

Leadership appears somewhat less effective in facilitating collusion when firms’ decision variables are strategic substitutes, as is often the case for quantity competition. In such a case, an aggressive move by the leader tends to induce the follower to reduce its own output, which may make it more difficult to punish such a move. In particular, quantity leadership does not enhance collusion when firms are quite impatient. Such leadership may however still increase collusive profits when firms are more patient.

The rest of the paper is organized as follows. Section 2 reviews the relevant economic literature. Section 3 shows how price leadership facilitates collusion in the case of Bertrand competition. Section 4 studies the same issue in the case of Cournot competition. Section 5 concludes.

2. Literature review

The literature identifies three types of leadership: dominant firm models, barometric and collusive price leadership. In the first type of situation, a large firm sets its price first and a smaller firm then typically matches this price. Such price leadership can occur for different reasons. For example, as Deneneck and Kovenock (1992) show, when two capacity-constrained firms compete in prices, the larger firm provides a price umbrella which allows the smaller firm to sell its capacity at a slightly better price. Another approach is provided by van Damme and Hurkens (1999, 2004), who develop a two-stage duopoly game where in each stage firms can choose a strategy (price or quantity). The game admits many equilibria but, applying the concept of risk dominance developed by Harsanyi and Selten (1988), the authors show that the unique risk dominant equilibrium is such that the more efficient firm acts as a leader.

Barometric leadership may arise in situations where some firms are better informed than others. Less informed firms may then delay their decisions until a better informed firm moves, thereby providing a signal about market conditions; the leader thus acts as a “barometer”. Such a model has recently been developed by Cooper (1997). He studies a two-stage duopoly game where in the first stage either firm can purchase information about the demand; the second stage then consists of T periods in each of which both firms can set their prices. Cooper shows that in the (unique) Bayesian equilibrium one firm purchases information and then moves first in the subsequent pricing stage.

While the literature on competitive price leadership is quite rich, few models have studied collusive price leadership. Markham (1951) is perhaps the first to address this issue. He argues that collusive price leadership can occur in lieu of overt collusion; i.e., if meetings in “smoked-filled rooms” violate antitrust laws, then firms may use public (pre-) announcements to coordinate on a collusive outcome.

Rotemberg and Saloner (1990) argue that asymmetric information may be the reason for collusive leadership to arise. They study an infinitely repeated game where in each period one firm perfectly knows the current state of demand while the other only knows the distribution of demand shocks. The authors show that there exist stationary equilibria in which the informed firm first sets its price while the uninformed one then matches this price.

Marshall et al. (2008) develop a model which attempts to replicate the pattern of price (pre-) announcements observed in the Vitamins cartel. Besides allowing firms to make pre-announcements, the authors introduce the mechanism of buyer “acceptance” of the pre-announced price increase. They show that, in contrast to the competitive regime in which firms only make announcements, in the collusive regime firms always use pre-announcements in order to test whether buyers accept a price increase.

The present paper identifies a different motivation for collusive leadership. Instead of focusing on coordination motives (as in Markham and Marshall et al.) or asymmetric information (as in Rotemberg and Saloner), it emphasizes the fact that collusive leadership may enhance the sustainability and thus the efficiency of collusion.

3. Price leadership as a facilitating device

In this section we show how leadership can facilitate collusion in various contexts of price competition.

3.1. Pure Bertrand competition

Consider first a simple Bertrand duopoly, where two identical risk neutral firms produce the same good with the same unit cost of production, c, and face a demand $q = D(p)$. If the firms could coordinate their decisions and share profits, they would jointly obtain the monopoly profit:

$$\Pi^M \equiv \max_p \Pi(p) \equiv (p-c)D(p).$$

In contrast, standard Bertrand competition yields prices set at cost and zero profits.

Suppose now that the two firms compete repeatedly over time and maximize the sum of their expected discounted profits:

$$(1-\delta) \sum_{t=0}^{\infty} \delta^t \pi_t(t),$$

where $\delta$ denotes the discount factor, which is assumed to be the same for both firms. To sustain collusion on a given price $p > c$, the best collusive strategies are then of the grim-trigger type, where both firms stick to the price $p$ as long as their rival does, and otherwise revert to the standard Bertrand equilibrium. Assuming that the firms share the market equally in equilibrium (which indeed facilitates collusion), such collusion is sustainable when the benefit from a short-term deviation 11

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4. While they exhibit collusive equilibria in which the informed firm acts as a leader, their analysis leaves open the question as to whether price leadership may still facilitate collusion in the absence of asymmetric information.

5. Marshall et al. use the terms “pre-announcement” and “announcement” in the following sense: a pre-announced price can be retracted prior to becoming effective, while an announced price cannot be retracted or altered and becomes immediately effective.

6. The static Nash equilibrium yields here mimimax profits and therefore constitutes a maximal—and thus optimal—punishment path.

11. We follow here the classic literature on supergames and tacit collusion, in which the “period” can be interpreted as the length of time during which firms cannot modify their strategies; a firm that “deviates” and undercut its rival will thus serve the entire market until the other firm is able to react. The duration of this period—reflected in the value of the discount factor—affects the profitability of the deviation and therefore the sustainability of collusion: the shorter the reaction delay, the less profitable the deviation and thus the easier it is to sustain collusion.
is more than offset by the loss of collusion in the future, that is, when and only when:

$$\frac{\Pi(p)}{2} \geq (1-\delta)\Pi(p) + \delta \times 0,$$

which boils down to:

$$\delta \geq \frac{1}{2}.$$

Therefore, perfect collusion \((\Pi(p) = \Pi^M)\) is sustainable whenever some collusion is sustainable, and collusion on any price \(p > c\) is sustainable if the discount factor is large enough, namely, if the discount factor is larger than a threshold equal here to \(1/2\).

We implicitly assumed so far that firms were required to set their prices simultaneously (we will refer to this game as the simultaneous pricing game). Suppose now instead that the firms can choose to set their prices sequentially. Formally, building on Hamilton and Slutsky (1990), we suppose that in each period the firms play the following competition game \(G\):

- **Stage 1:** Each firm can set its price, which is then publicly observable.
- **Stage 2:** Any firm that has not done so in Stage 1 now sets its price, which is again publicly observable. Demand is then realized and profits are obtained.

This competition game thus allows the firms to set prices either simultaneously or sequentially; as in the case of simultaneous moves, once a price has been set, it cannot be withdrawn or altered. Stage 1 can be interpreted as the possibility to declare its price "in advance" for the coming period. In practice, prices may be set well before the relevant period; what matters here is that firms can set their prices at different times for the same period.

We also assume that, when both firms charge the same price, they can share the market as they wish. This assumption (which does not affect the analysis presented so far) can be interpreted as the limit case of negligible product differentiation, where the firms could indeed share the market as they wish through a small price difference. Later on, we extend the model to account for cost asymmetry and product differentiation.

Note that the set of equilibria can only increase when moving from the simultaneous pricing game to the game \(G\). Indeed, consider an equilibrium of the repeated simultaneous pricing game, and suppose that the firms adopt the same strategies in the first stage of the repeated game \(G\) (that is, in each period they set their prices in Stage 1 as they would in the repeated simultaneous pricing game, given the past history of prices). These strategies still sustain an equilibrium in the repeated game \(G\), since deviating to any given price \(p\) (in either stage) yields the same payoff as in the repeated game with simultaneous moves.

We now show that the firms can collude in the repeated game \(G\) even when they could not do so with simultaneous pricing. To see this, consider the following strategies:

- One firm, acting as a leader, sets its price to the monopoly level in Stage 1 and the other firm, acting as a follower, then matches this price in Stage 2; the leader obtains a market share \(\alpha_t \in [0, 1]\) while the follower obtains \(\alpha_t = 1 - \alpha_t\).
- Any deviation by either firm is followed by reverting forever to the static Bertrand outcome in the following periods; in addition, if the leader deviates in Stage 1, then in Stage 2 the follower slightly undercut the leader if it set a price above cost in Stage 1, or else prices at cost (if the leader did not set a price in Stage 1, then the firms play the static Bertrand equilibrium in Stage 2).

By construction, no firm has an incentive to deviate from any punishment path, since they then systematically "best respond" to each other (in particular, if the leader deviates in Stage 1, then the follower best responds myopically in Stage 2 before they revert to the static Bertrand equilibrium).14 Clearly, the leader cannot gain either from deviating from the equilibrium either, since all deviations yield zero profit. Finally, the follower cannot gain from deviating from the equilibrium path if and only if:

$$\alpha_t = 1 - \alpha_t \leq \delta_t \equiv \delta.$$

Therefore, for any \(\delta \in (0, 1)\) and \(\alpha \in [0, \delta]\), there exists an equilibrium in which the leader gets a share \(\alpha\) and the follower gets a share \(1 - \alpha\) of the monopoly profit.15

The following proposition summarizes the above analysis.

**Proposition 1.** When the firms must set their prices simultaneously, they can collude only when \(\delta\) is large enough, namely \(\delta \geq 1/2\). In contrast, when firms have the possibility of setting their prices sequentially, then by doing so they can collude and maintain the monopoly price for any discount factor \(\delta \geq 0\).

Price leadership thus arises endogenously in our framework: even though the firms could still choose to set prices simultaneously, they can find it profitable to adopt sequential pricing in order to facilitate collusion. Price leadership allows the firms to collude even when they could not do so with simultaneous pricing; since the leader has less incentive to deviate (no incentive, actually), it suffices to grant a large enough market share to the follower to restore collusion.

**Remark 1.** To endogenize the timing of the pricing decisions, the game \(G\) follows the approach developed by Hamilton and Slutsky, in which firms can choose whether to set their prices for the coming period simultaneously or sequentially. One may however question the underlying assumptions of this approach, and in particular the ability of the firms to commit for exactly one and only one period. In practice, firms may be free to choose the period of validity of their prices, and may moreover choose when to set their prices. In the absence of any restriction on the frequency of pricing decisions, revising these decisions "in turn", but infinitely quickly, would then allow the firms to maintain monopoly prices and profits even without any "collusive" scheme: indeed, in their analysis of dynamic oligopolies, Maskin and Tirole (1988) show that, when firms take turns to set prices, there exists a Markov perfect equilibrium (with a "kinked demand" flavor) that yields the monopoly outcome whenever the discount factor is close enough to 1, which can be interpreted as a high frequency of pricing decisions. Our analysis focuses instead on markets where firms cannot change their

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14 Since a deviation by the leader leads to reversal to Nash in the following periods, whatever the follower does, the follower is indeed willing to best respond by slightly undercutting the leader in the current period. Conversely, this reaction of the follower suffices to deter any deviation by the leader; thus, we could as well assume that, once the follower has "best responded", the firms revert to collusion in the remaining periods.

15 Formally speaking, even when the competition game is not repeated (that is, \(\delta = 0\)), there exist an equilibrium in which the leader charges the monopoly price, and the follower matches the price and gets the entire market—and any other price \(p = \frac{c}{2}\) could be supported in this way. This equilibrium is however rather knife-edge: it disappears, for example, if delaying the price decision until stage 2 involves an arbitrarily small cost (e.g., losing a few customers); the follower would then deviate and match the rival's price in stage 1. In contrast, introducing such an infinitesimal cost would not destroy the collusive equilibria when the game is repeated and \(\delta > 0\).

16 Hamilton and Slutsky actually consider two variants, the one reflected in game \(G\) and another one where the firms first (simultaneously) decide in which stage they will play.
prices too often: as in other standard models of collusion, the generic “period” of the game $G$ can then be interpreted as the minimal amount of time during which prices cannot be changed once they are set. We build on these standard models by allowing however firms to choose whether to set these prices at the same time or with an (arbitrarily small) time lag.17

**Remark 2.** We have considered so far equilibria in which the same firm acts systematically as a leader; as a result, the benefits from collusion may be shared somewhat asymmetrically, as the follower needs to be given a large enough market share when the discount factor is low. The firms could however adapt the above scheme in different ways in order to share the gains from collusion in a more balanced way. For example, they could randomly draw who will be the leader. Then, while ex post the unlucky firm that ends up being the leader may obtain a smaller share of the collusive gains, ex ante the expected gains could be divided equally. Alternatively, if transfers are feasible the firm selected to be the follower could compensate the other firm for the market share asymmetry. Exactly how effective this can be will depend on the nature of transfers and the distortions they may generate, their visibility asymmetry. Exactly how effective this can be will depend on the nature of transfers and the distortions they may generate, their visibility asymmetry. Exactly how effective this can be will depend on the nature of transfers and the distortions they may generate, their visibility asymmetry.

Many industries involve segmented markets, in which case the firms can play different roles in different segments. For example, Harrington (2006) notes that, in the case of global cartels, “the firm that took the lead in changing prices could vary across countries”. This allows the firms to facilitate collusion and to share the gains more evenly. Suppose for example that firm 1 acts as a leader on market segments representing half of the entire market (and gets a share $\alpha_1$ of each of these segments), the other firm acting as a leader in the other half. If a firm wants to deviate, the best deviation consists in undercutting its rival in Stage 2 in those segments where it acts as a follower, and not deviating in Stage 1 where it acts as a leader: in this way, the firm obtains the entire monopoly profits where it is a follower, and still earns a share $\alpha_2$ of these profits where it is a leader. Each firm will therefore refrain from deviating if and only if:

$$
\frac{\Pi^M}{2} \geq \left(1 - \delta\right) \frac{1}{2} + \alpha_2 \Pi^M + \delta \times 0,
$$
or:

$$
\alpha_2 = 1 - \alpha_1 \leq \alpha_1 \equiv \frac{\delta}{1 - \delta}.
$$

Therefore:

- distributing leadership symmetrically across market segments makes again collusion sustainable even when it would not be so with simultaneous pricing, i.e., when $\delta < 1/2$; in that case, firms can indeed sustain collusion by granting a share $\alpha_\text{L} \equiv \left[0, \alpha_1\right]$ to the leader—who may well have here a larger share than the follower, since $\alpha_\text{L} > 1/2$ as long as $\delta > 1/3$;
- in each segment, collusive market shares need to be less asymmetric than before $\left(\alpha_\text{L} \cdot \alpha_\text{L}\right)$—if $\delta > 1/3$, each segment may actually be shared equally; and some asymmetry may be required at the segment level (if $\delta < 1/3$), this scheme allows the firms to achieve overall symmetric market shares and profits.

Firms could also share the gains from collusion by alternating the roles or randomly selecting the leader in each period. In particular, when the market is segmented, firms could have stable leaders in some segments and rotating leaders in others.19 In theory, taking turns could still facilitate collusion even in the absence of any market segmentation. If for example the firms toss a coin to select the leader at the beginning of each period, the follower will then refrain from deviating if and only if:

$$
(1 - \delta)\alpha_1 \Pi^M + \frac{\Pi^M}{2} \geq (1 - \delta) \Pi^M + \delta \times 0,
$$
or:

$$
\alpha_1 = 1 - \alpha_1 \leq \alpha_1 \equiv \frac{\delta}{2(1 - \delta)}.
$$

Randomly selecting a leader thus makes again collusion sustainable even when $\delta < 1/2$; in that case, firms should however grant a smaller market share to the leader $\left(\alpha_1 \leq \alpha_1\right)$, where $\alpha_1 < 1/2$ whenever $\delta < 1/2$, implying that market shares, and not only the identity of the leader, would vary significantly over time, a feature which we do not seem to observe in practice in discovered cartels.

### 3.2. $n$-firm oligopoly

We briefly extend here the analysis readily to the case where $n > 2$ firms compete in the market. Increasing the number of competitors moreover enriches the way sequence of the pricing decisions can interact with product market competition. To give only a few examples, $l$ leaders could be followed by the other $n - l$ firms, each firm could indeed set its own price in sequence, and so on. For our analysis, however, what matters is that followers are in a good position to punish more effectively those firms that are expected to set prices beforehand. In particular, the previous analysis readily extends to the case where a single follower sets its price after all the others: whether the others set their prices simultaneously or in sequence is then irrelevant, since any deviation by one of these firms will be immediately sanctioned by the last follower, and thus cannot be profitable. Therefore, collusion can again always be sustained, by granting a large enough market share—that is, satisfying (1)—to the last follower.

The analysis can also be extended to the case of multiple last followers, but the sustainability condition (1) must be satisfied for each of them; as a result, collusion with $f$ followers is sustainable only if the discount factor exceeds $1 - 1/f$.

### 3.3. Asymmetric costs

We now show that the previous insight is robust to the introduction of cost asymmetry and consider the same price duopoly as before, except that firm 1, say, is now more efficient than its rival: $c_1 \leq c_2$. The profit that firm 1 could achieve is therefore $\Pi_1 = (p - c_1)D(p)$. Let $p^M$ and $\Pi^M$ denote, correspondingly, the monopoly price and the monopoly profit for firm 1. We assume that the cost difference is not drastic, so that the less efficient firm can still exert a competitive pressure on its rival: $c_2 > p^M$.

17 This corresponds for example to markets where sellers publish price catalogs, which would be costly to modify once they have been circulated. Firms may however decide to publish these catalogs more or less in advance. See De Neckere and Kovnen (1992) at pp. 151–153 for a related discussion.

18 Suppose for example that the follower can make a transfer $T = \left(c_1 - 1/2\Pi^M\right)$ at the beginning of each period, before the leader’s pricing decision. Then, price leadership allows to sustain collusion and to share the gains evenly; in particular, making the transfer is indeed sustainable: if for example any deviation at that stage leads the firms to implement the Bertrand outcome in the second stage of each period (including the deviating period), the sustainability condition boils down to

$$
T + \alpha_1 \Pi^M = \Pi^M / 2 \geq (1 - \delta) \times 0 + \delta \times 0 = 0.
$$

which is trivially satisfied.

19 In the case of the graphite electrode cartel, for example, Harrington [1999] notes that the market leaders SGL and UCAR followed different rules in the various European markets. In “Italy and Spain, where they had roughly equal market shares, they decided on each occasion which one of them would ‘publicly’ act as the price leader to get the new price established in the market”. In contrast, SGL was the price leader in Germany and Scandinavia, where it had a more important presence, whereas UCAR acted as price leader in France and United Kingdom.
As is now well-known, cost asymmetry reduces the scope for collusion. As pointed out by Harrington (1991), the critical threshold for the discount factor that allows collusion with grim-trigger strategies then exceeds 1/2. Miklos-Thal (2011) shows that, while maximal punishments involve zero continuation payoffs and allow for some collusion whenever \( \alpha > 1/2 \), the set of collusive prices and market shares remains smaller than when firms face symmetric costs.

Using a similar reasoning as before, price leadership still makes collusion sustainable even when it would not be so with simultaneous pricing, i.e., when \( \alpha < 1/2 \). To see this, suppose that the less efficient firm acts as a leader and adapt the above punishment strategies as follows: any deviation by either firm is followed by maximal punishment in the following periods; and if the leader deviates in Stage 1, then in Stage 2 the follower slightly undercuts the leader (if it set a price above the follower’s cost) or prices at cost (if the leader set no price in Stage 1, then the firms play the static Nash in Stage 2). By construction, these punishment strategies constitute equilibrium, which moreover deter the leader from deviating and yield maximal punishment for the follower. As before, granting a large enough market share to the follower then suffices to deter it as well from deviating and, therefore, to sustain collusion.

We now show that, even when the firms could collude with simultaneous pricing, they can still collude in a better way with sequential pricing. As we will see, price leadership actually allows both firms to increase their collusive profits, although this requires the less efficient firm to act as the leader.

In the absence of sustainability constraints the Pareto optima, from the point of view of the firms, consist of a price \( p^M \in [p_1^M, p_2^M] \) and a market share \( \alpha(c') \), which decreases with \( c' \), from \( \alpha = 1 \) for \( c' = p_1^M \) (this corresponds to the monopoly outcome for the efficient firm) to \( \alpha = 0 \) for \( c' = p_2^M \) (corresponding to the monopoly outcome for the other firm). To be sustained, however, they must resist deviations by either firm.

In the case of simultaneous pricing, and since \( p' \geq p_2^M \), firm 1’s best deviation involves setting \( p_1 = p_1^M \), since maximal punishments involve zero profit, the incentive condition writes as:

\[
\alpha(c_1 \Pi_1(p') \geq (1-\delta)\Pi_1^M + \delta \times 0.
\]

or:

\[
\alpha(c_1 \Pi_1(p') \geq (1-\delta)\Pi_1^M / \Pi_1(p') \]

IC1

In contrast, since \( p' \leq p_2^M \), firm 2’s best deviation simply consists in slightly undercutting its rival; its incentive constraint:

\[
\alpha(c_2 \Pi_2(p) \geq (1-\delta)\Pi_2(p') + \delta \times 0.
\]

thus boils down to:

\[
\alpha(c_2 \Pi_2(p) \leq \delta.
\]

IC2

Miklos-Thal (2011) moreover shows that this latter constraint (IC2) is the one that may prevent collusion from further enhancing both firms’ profits.²¹ The intuition is straightforward and relies on the observation that (IC1) coincides with firm 1’s iso-profit curve. Suppose that a constrained Pareto optimal collusive outcome \( (\alpha(c_1, p') \) (that is, an outcome that is Pareto efficient for the firms, among those that satisfy (IC1) and (IC2)) does not constitute an unconstrained optimum. By construction, it is then possible to find an alternative outcome \( (\alpha(c_1, p') \) satisfying \( \alpha(c_1 \Pi_1(p') > \alpha(c_1 \Pi_1(p') + (1-\alpha(c_1) \Pi_1(p') > (1-\alpha(c_1) \Pi_1(p') \)

But then, \( \alpha(c_1 \Pi_1(p') > \alpha(c_1 \Pi_1(p') \geq (1-\delta)\Pi_1^M \), and thus the outcome \( (\alpha(c_1, p') \) not only Pareto improves upon \( (\alpha(c_1, p') \) but also satisfies (IC1); it must therefore violate (IC2).

Suppose now that the firms repeat the competition game G. As already noted, they can still sustain the same equilibria as with simultaneous pricing (by charging the same prices in the first stage of game G). But price leadership moreover allows the firms to increase both firms’ profits whenever simultaneous pricing does not enable them to achieve an unconstrained optimum: since price leadership relaxes the incentive constraint of the leader, and it is the less efficient firm’s incentive constraint that can prevent the firms from achieving an unconstrained Pareto optimum, it suffices to have this firm acting as a leader.

**Proposition 2.** Whenever simultaneous pricing does not allow firms to achieve an unconstrained Pareto optimum, price leadership allows them to increase both of their profits from collusion, provided that the less efficient firm acts as a leader.

**Proof.** See Appendix B. □

Price leadership may also enhance collusion in more complicated settings. For example, Athey and Bagwell (2001) consider a situation where in each period the firms receive (privately observed) cost shocks; they show that optimal collusion requires asymmetric, history-contingent strategies: the less efficient firm should abstain from production whenever their costs differ, and when they face the same cost a firm that did not produce in the previous period should get a larger market share. In the first case, the less efficient firm has stronger incentives to deviate, while in the second case it is the firm with a lower market share that is more tempted to deviate. In the absence of transfers, the only way to induce them to abide to the collusive agreement is to reward them in the future, which is possible only when the firms are sufficiently patient. When the firms are less patient, the incentive constraint of the less efficient firm or of the firm with the lower market share is binding, which limits the scope for collusion. Allowing the firms to move in sequence could again help; however, the firms should then select the leader as a function of the past history, so as to relax the most stringent incentive constraint.

### 3.4. Differentiated products

We now extend the analysis to account for product differentiation, and consider a symmetric duopoly model where the firms face the same cost of production, \( C(q) \), and demand functions of the form, for \( i \neq j = 1, 2 \), \( D_i(p_i, p_j) = D(p_i, p_j) \), which respectively decrease and increase with respect to their two arguments. In a given period, the profit of firm i is \( \pi(p_i, p_j) = p_i D(p_i, p_j) - C(D(p_i, p_j)) \). We will assume that these profit functions are "well-behaved", namely:

- Joint profits are maximal for symmetric prices; we will denote by \( p^M = \arg\max_{p, p'} \pi(p, p) \) and by \( \pi^M = \pi(p^M, p^M) \) the monopoly and per-firm profit.
- Profit functions are twice differentiable and strictly concave (i.e., \( \partial^2 \pi > 0 \); implying that best responses are well-defined); prices are strategic complements (i.e., \( \partial^2 \pi > 0 \)) and one-shot simultaneous pricing game has a unique stable Nash equilibrium (i.e., best responses have a slope that is positive but lower than unity), which

²² Throughout the paper, we denote by \( \partial f \) the partial derivative of the function \( f \) with respect to its ith argument.
is symmetric; we will denote by \( p^N \) and \( \pi^N = n(p^N, p^N) \) the Nash equilibrium price and profit.

- There is also a unique subgame perfect equilibrium in the one-shot Stackelberg game; we will denote by \( p^F \) and \( \pi^F = n(p^F, p^F) \) the equilibrium prices, and by \( \pi^N = n(p^N, p^N) \) the resulting profits for the leader and the follower. The above assumptions imply \( \pi^N > \pi^F > \pi^N \) (see Appendix C).

Suppose first that the firms play the simultaneous pricing game repeatedly over time. The most profitable symmetric pure strategy equilibria are of the form: in every period, both firms set the same price, \( p \), and any deviation triggers a switch to the worst punishment continuation equilibrium. Such collusion is sustainable if and only if:

\[
\max_{p'} \pi(p', p) - \pi(p, p) \leq \frac{\delta}{1-\delta} \pi(p, p) - \zeta,
\]

where \( \zeta \) is the worst sustainable payoff. From now on we will denote by \( p^* \) the best sustainable price with simultaneous pricing; that is,

\[
p^* = \min\{p, p^M\},
\]

where \( p = p^* \) is the highest price for which Eq. (2) is binding. Similarly, we will denote by \( \pi^N (p, p^*) \) the corresponding profit for each firm.

Let us now introduce sequential pricing. If the firms play the competition game \( G \) once, there exist three equilibria in pure strategies:

- one in which both firms set \( p^0 \) in Stage 1, and two in which each firm sets \( p^0 \) in Stage 1, while its rival sets \( p^2 \) in Stage 2.

Therefore, as long as \( \pi^N < \pi^F < \pi^M \), and thus price leadership cannot generate greater collusive profits. Finally, when \( \pi^{N} < \pi^{T} / 2 < \pi^{N} / 2 \), price leadership could first enhance collusion through enhanced punishments, i.e., by relying on sequential pricing off the equilibrium path; but even ignoring this possibility, adopting a price leader along the equilibrium path already suffices to enhance collusion:

**Proposition 3.** Whenever the firms cannot sustain the monopoly profit with simultaneous pricing, selecting one firm as a systematic price leader allows them to increase their joint collusive profit.

**Proof.** See Appendix C. □

To see how price leadership allows the firms to sustain greater profits, note that: (i) as before, price leadership relaxes the leader’s incentive constraint, which is thus no longer binding at \((p^*, p^*)\); and (ii) a small increase in the leader’s price above \( p^* \) then relaxes as well the follower’s incentive constraint, as it reduces its short-term gain from deviation and increases its future profit from collusion.

While price leadership allows the firms to increase in this way their joint profit, it also creates an asymmetry among the two firms’ profits, the follower generally enjoying a greater share of these collusive profits. Symmetry could again be restored in various ways, e.g. by “taking turns” or by randomly selecting ex ante the leader.

Even in the absence of such rebalancing scheme, collusion may allow greater profits for both firms when the scope for collusion would otherwise be limited. This is obvious when \( \pi^N < \pi^F < \pi^M \), since in that case simply replicating the Stackelberg outcome does the trick; a continuity argument extends this insight to larger values for \( \pi^N \):

**Proposition 4.** There exists \( p_f \) satisfying \( \pi^N < p_f < \pi^M \) such that, whenever \( \pi^N < p_f \), selecting one firm as a systematic price leader allows both firms to obtain more than \( \pi^N \) in every period.

**Proof.** See Appendix D. □

### 4. Cournot competition

We now turn to the case of quantity competition assuming that the firms play the same two-stage game except that they set quantities rather than prices. To keep in line with our previous analysis, we will assume that once a quantity has been set it cannot be changed afterwards within the same period. This can for example be the case when firms need to plan their capacities in advance, and departing from plan involves considerable costs.

In contrast to Bertrand competition when prices are usually strategic complements, in Cournot competition quantities are usually strategic substitutes. This has several consequences. For example, in contrast to the Bertrand case, in a static framework the presence of a leader reduces industry profit since the leader takes advantage of its position by behaving more aggressively. By the same token, in a repeated competition framework quantity leadership may fail as well to enhance collusion, particularly when firms are rather impatient. To see this, consider a symmetric duopoly where the firms share an inverse demand function \( p = P(q) \) and face the same cost \( C(q) \), and let \( n(q, q) \) denote the corresponding duopoly profit:

\[
n(q, q) = P(q + q) - C(q).
\]

We will assume the following regularity conditions:

- The demand function \( P(q) \) is downward-sloping, the cost function \( C(q) \) is convex and the function \( n(q, q) \) is single-peaked.
- The profit function \( n \) is twice differentiable and there exists \( B, b > 0 \) such that, \( \forall q, q', \partial_q n(q, q) \leq -B \) and \( \partial_q n(q, q) \geq -b \);
- Quantities are strategic substitutes and there exists a unique, stable Cournot (static Nash) equilibrium, \((q^*, q^*)\).

These properties first imply that in the monopoly outcome the two firms produce the same quantity,\(^{24}\) which we will denote by \( q^M \) and is uniquely characterized by \( q^M = \arg\max q C(q, q) \). They also ensure that the best response \( r(q) = \arg\max q n(q, q') \) is well defined; strategic substitutability then amounts to \( r(q) < 0 \), or \( \partial_q r(q) < 0 \), while stability requires \( r(q) > 0 \), or \( \partial_q r(q) > 0 \). These conditions are satisfied in standard models (e.g., linear demand and cost) and imply that the Stackelberg outcome is less profitable than the (symmetric) Nash equilibrium. But for low values of the discount factor, in any equilibrium in which a leader sets its quantity before its rival, the follower is deemed to set a quantity that is close to the static best response, which in turn induces the leader to choose a quantity close to the static Stackelberg level. Therefore, total

\[\]

\(^{24}\) The industry profit \( \Pi(q, q) = n(q, q) + n(q, q) \) satisfies

\[
\Pi(q, q) = P(q+q)(q+q)-C(q_q)-C(q_q) = 2qP(q)C(q)-C(q) \geq 2B(P(q)C(q)-C(q)) \]

where \( q = (q+q)/2 \) and the inequality follows from the convexity of the cost function \( C \). Therefore, any asymmetric outcome is (at least weakly) dominated by a symmetric one; if \( C \) is strictly convex, the monopoly outcome is moreover unique and symmetric. In case of linear costs, there can exist several outcomes in which the two firms share the same total output; symmetric market shares are however easier to sustain in collusion with simultaneous moves.

---

\(^{23}\) See Hamilton and Slutsky (1990).
profit is close to the Stackelberg one, and thus is lower than that of the simultaneous move Nash equilibrium. This yields:

**Proposition 5.** Quantity leadership cannot enhance collusion when the discount factor is small enough.

**Proof.** See Appendix E. □

We now show that even ignoring the possibility that quantity leadership may help inflict tougher punishments, it may still facilitate collusion when firms are less impatient (but not too patient either, since otherwise they could sustain the monopoly outcome even without leadership), by making it easier to punish the leader in case it deviates.

Suppose first that firms are required to set their quantities simultaneously; the most profitable symmetric collusion is then of the form: "set \( q \) as long as your rival does so, otherwise switch to the worst sustainable punishment". Letting \( w \) denote this punishment, a quantity \( q \) is then sustainable if and only if:

\[
\max_{q'} \pi(q', q) - \pi(q, q) \leq \frac{6}{1-\delta} [\pi(q, q) - w].
\]  (3)

We will denote by \( q^* \) the best symmetric collusive output:

\[
q^* = \max \{ q, q^M \},
\]

where \( q \) is the lowest quantity \( q \) for which Eq. (3) binds. Likewise, let \( \pi' = \pi(q^*, q^*) \) and \( \pi^M = \pi(q^M, q^M) \) denote the corresponding profits per firm.

Suppose now that in each period the same firm, acting as a leader, sets \( q^L \) in Stage 1 while the follower sets \( q^F \) in Stage 2. The leader thus earns \( \pi(q^L, q^F) \) while the follower earns \( \pi(q^F, q^L) \). As before, deviations from the follower can be punished by the worst punishment available with simultaneous moves, \( w \): the follower then refrains from deviating if and only if:

\[
\max_{q'} \pi(q^L, q') - \pi(q^F, q') \leq \frac{6}{1-\delta} [\pi(q^F, q^L) - w].
\]  (4)

As for the leader, we will adapt the previous type of punishment as follows:

- if in Stage 1 the leader sets an unexpected quantity, then in Stage 2 the follower best responds (in a "static way") to that quantity;
- if instead the leader does not set a quantity in Stage 1, then in Stage 2 the firms play the static Nash equilibrium;
- in both cases, in the remaining periods (even if the follower does not react as expected), the firms switch to the continuation equilibrium that inflicts the worst sustainable punishment \( w \) (in simultaneous moves) to the leader.

Deviating in a given period then cannot give the leader more than the static Stackelberg profit, which we will denote again by \( \pi^* \). The leader will thus refrain from deviating if and only if:

\[
\pi^* - \pi(q^L, q^F) \leq \frac{6}{1-\delta} [\pi(q^F, q^L) - w].
\]  (5)

The following proposition shows that quantity leadership can enhance collusion when it would already be somewhat effective in a simultaneous move setting:

**Proposition 6.** Suppose \( \pi' = \max \pi(q, r(q), q^M) < -\pi^M = \max \pi(q, q^M) \). Then there exists a threshold \( \Pi^* \) such that, when \( \pi' \geq (\Pi^*, \pi^M) \), selecting one firm as a systematic quantity leader allows the firms to increase their joint collusive profit.

**Proof.** See Appendix E. □

Intuitively, relaxing the follower’s constraint (Eq. (4)) requires allocating a larger market share to that firm, which however tends to exacerbate the leader’s incentive to deviate. Since the punishment in the following periods is here the same as with simultaneous moves, leadership enhances collusion only if the leader’s short-term gain from a deviation, \( \pi^* \), is lower than that with simultaneous moves. Proposition 5 implies that this cannot be the case when the discount factor is low; indeed, the collusive outcome with simultaneous moves, \( (q^*, q^*) \), is then close to the static Nash outcome, which implies that the short-term gain from deviation, \( \max \pi(q^*, q^*) \), is itself close to \( \pi^* \) and therefore lower than \( \pi^* \). In contrast, when firms are more patient, \( q^* \) is closer to the monopoly quantity \( q^M \), the condition \( \pi^* < \pi^M \) (which is for example satisfied in the linear Cournot model25) then ensures that limiting the leader’s short-term gain from deviating to \( \pi^* \) provides a more effective deterrent than in the case of simultaneous moves, where the rival sticks to \( q^* \) in the deviating period.26 Quantity leadership then relaxes the leader’s incentive constraint and thus allows the firms to increase their aggregate profit from collusion.

While quantity leadership may allow the firms to increase their joint profit, it again creates an asymmetry among the two firms’ profits; indeed, the proof of the above proposition relies on collusive schemes in which the follower obtains a slightly larger market share. Such asymmetry is not a problem for retaliation strategies, however, since the purpose is to punish the deviator: quantity leadership may therefore be used to enhance punishments. For example, when the discount factor is low, simultaneous moves may only allow for punishment profits close to \( \pi^* \), while simply reverting to the static Stackelberg equilibrium27 suffices to impose a tougher punishment, \( w = \pi^* - \pi^* \), on the follower.

By contrast, adopting quantity leadership along the equilibrium path creates an asymmetry in collusive profits. Additional mechanisms, such as randomly selecting the leader ex ante, then need to be used to maintain a balanced division of the gains from collusion. In the absence of such rebalancing schemes, it is more difficult, if not impossible, to ensure that both firms maintain collusion from quantity leadership. Indeed, in the linear case we have:

**Proposition 7.** If \( P(q) = d - q \) and \( C(q) = cq \) then, keeping constant the punishment level \( w \leq \pi^* \), one firm systematically acting as leader does not allow both firms to achieve greater collusive profits than \( \pi^* \).

**Proof.** See Appendix G. □

Thus, in contrast to price leadership, quantity leadership along the equilibrium path does not allow the firms to achieve a Pareto improvement.28 Intuitively, to increase their profits the firms should lower their outputs. But since quantities are strategic substitutes here, if the leader reduces its output then the follower should increase its own quantity in order to sustain collusion; the proposition confirms that no outcome satisfies both conditions.

This finding is in line with empirical observations. In our survey of EC cartel decisions, while (production or distribution) capacity appears as the key strategic variable in 5 decisions,29 leadership does...
not feature in any of these decisions; in contrast, price leadership features in 16 of the other 44 decisions.

5. Conclusion

The above analysis shows that, in a setting where firms can move either simultaneously or sequentially, (price or quantity) leadership facilitates collusion. The intuition is that having one firm as a leader relaxes its incentives to deviate and increases in this way the scope for collusion.

In the case of price competition, we find that the leader has no incentives to defeat the cartel agreement. Rotemberg and Saloner (1990) provide a similar insight, although for a different reason. In their approach the follower, being uninformed, has no means to detect deviations by the leader (which is more informed about market conditions). The leader then faces no punishment, whatever price it announces, and thus always picks its most preferred price. In our approach, deviations by the leader are instead perfectly detected, but then immediately punished.

In contrast to the approach taken by Marshall et al. (2008) where firms use pre-announcements for the purpose of coordination on the collusive outcome, our approach instead focuses on the sustainability and efficiency of collusion. Moreover, as the authors note, their model does not require pre-announcements to be made sequentially (simultaneous pre-announcements make no difference). In contrast, we show that firms may wish to sequence the setting of their prices so as to relax sustainability constraints.

Besides illustrating the advantages of collusive leadership, the paper delivers several policy implications. First, our model predicts that leadership is a more effective collusive device when firms compete in prices rather than in quantities; this prediction is in line with the finding of our survey of EC cartel decisions. Second, when firms differ in their costs then the cartel leader is likely to be the less efficient firm. Finally, regardless of whether firms compete in prices or quantities, in order to facilitate collusion the firm acting as a follower in a given period should get a higher market share and earn a higher profit in that period.

Appendix A. Summary of EC decisions

We present here the findings of a survey of the EC cartel decisions. We considered all the public versions of final decisions that were made available online in English on the website of DG Competition (http://ec.europa.eu/competition/cartels/cases/cases.html) on July 21, 2010, which represent 49 cases. For each case, we first assessed whether some form of price (including interest rate, commission, and so forth) or quantity (including production or distribution capacity) appeared as the key strategic variable. We then looked for instances of leadership in the announcements of these strategic variables (when such occurrences were found, we mention in parenthesis the corresponding paragraph of the decision).

<table>
<thead>
<tr>
<th>Case</th>
<th>Decision date</th>
<th>Nature of competition</th>
<th>Leadership</th>
</tr>
</thead>
<tbody>
<tr>
<td>COMP/39.129—Power Transformers</td>
<td>2009</td>
<td>Price</td>
<td>No</td>
</tr>
<tr>
<td>COMP/39406—Marine Hoses</td>
<td>2009</td>
<td>Price</td>
<td>No</td>
</tr>
<tr>
<td>COMP/39125—Carglass</td>
<td>2008</td>
<td>Price</td>
<td>No</td>
</tr>
<tr>
<td>COMP/39188—Bananas</td>
<td>2008</td>
<td>Price</td>
<td>No</td>
</tr>
<tr>
<td>COMP/39181—Candle Waxes</td>
<td>2008</td>
<td>Price</td>
<td>Yes (113)</td>
</tr>
<tr>
<td>COMP/38.695—Sodium Chlorate</td>
<td>2008</td>
<td>Price</td>
<td>No</td>
</tr>
<tr>
<td>COMP/39165—Flat glass</td>
<td>2007</td>
<td>Price</td>
<td>Yes (211,214)</td>
</tr>
<tr>
<td>COMP/38.412—Professional Videotape</td>
<td>2007</td>
<td>Price</td>
<td>Yes (63)</td>
</tr>
<tr>
<td>COMP/38710—Bitumen Spain</td>
<td>2007</td>
<td>Price</td>
<td>No</td>
</tr>
<tr>
<td>COMP/E-1/39.168—Fasteners</td>
<td>2007</td>
<td>Price</td>
<td>Yes (93)</td>
</tr>
<tr>
<td>COMP/E-1/38.823—Elevators and Escalators</td>
<td>2007</td>
<td>Price</td>
<td>No</td>
</tr>
<tr>
<td>COMP/F/38.899—GAS INSULATED SWITCHGEAR</td>
<td>2007</td>
<td>Price</td>
<td>No</td>
</tr>
<tr>
<td>COMP/F/38.636—Synthetic rubber</td>
<td>2006</td>
<td>Price</td>
<td>No</td>
</tr>
<tr>
<td>COMP/F-1/38.121—FITTINGS</td>
<td>2006</td>
<td>Price</td>
<td>Yes (135,160)</td>
</tr>
<tr>
<td>COMP/38.456—Bitumen—NL</td>
<td>2006</td>
<td>Price</td>
<td>No</td>
</tr>
<tr>
<td>COMP/F/38.645—Methacrylates</td>
<td>2006</td>
<td>Price</td>
<td>Yes (104)</td>
</tr>
<tr>
<td>COMP/F/38.620—Hydrogen Peroxide</td>
<td>2006</td>
<td>Quantity</td>
<td>No</td>
</tr>
<tr>
<td>COMP/F/38.443—Rubber chemicals</td>
<td>2005</td>
<td>Price</td>
<td>Yes (63)</td>
</tr>
<tr>
<td>COMP/38354—Industrial bags</td>
<td>2005</td>
<td>Price</td>
<td>No</td>
</tr>
<tr>
<td>COMP/39337/E1/PO/Thread</td>
<td>2005</td>
<td>Price</td>
<td>No</td>
</tr>
<tr>
<td>COMP/C.38.281/B.2—Raw Tobacco Italy</td>
<td>2005</td>
<td>Price</td>
<td>No</td>
</tr>
<tr>
<td>COMP/E-1/37.773—MCAA</td>
<td>2005</td>
<td>Price</td>
<td>Yes (88)</td>
</tr>
<tr>
<td>COMP/E-2/37.533—Choline Chloride</td>
<td>2004</td>
<td>Price</td>
<td>No</td>
</tr>
<tr>
<td>COMP/E-1/38.069—Copper Plumbing Tubes</td>
<td>2004</td>
<td>Price</td>
<td>Yes (205,206)</td>
</tr>
<tr>
<td>COMP/C.37.750/B2—French beer</td>
<td>2004</td>
<td>Quantity</td>
<td>No</td>
</tr>
<tr>
<td>F-1/38.318—PO/Needles</td>
<td>2004</td>
<td>Price</td>
<td>No</td>
</tr>
<tr>
<td>COMP/C.38.238/B.2, Raw Tobacco Spain</td>
<td>2004</td>
<td>Price</td>
<td>No</td>
</tr>
<tr>
<td>C.38.359. Electrical and graphite products</td>
<td>2003</td>
<td>Price</td>
<td>Yes (101)</td>
</tr>
<tr>
<td>COMP/E-1/38.240. Industrial tubes</td>
<td>2003</td>
<td>Price</td>
<td>No (58)</td>
</tr>
<tr>
<td>COMP/E-2/37.857—Organic Peroxides</td>
<td>2003</td>
<td>Price</td>
<td>No</td>
</tr>
<tr>
<td>COMP/E-1/37.370. Sorbates</td>
<td>2003</td>
<td>Price</td>
<td>Yes (103)</td>
</tr>
<tr>
<td>COMP/E-2/37.667—Speciality Graphite</td>
<td>2002</td>
<td>Price</td>
<td>No</td>
</tr>
<tr>
<td>COMP/E-1/37.152—Plasterboard</td>
<td>2002</td>
<td>Price</td>
<td>Yes (203,291)</td>
</tr>
<tr>
<td>COMP/E-2/37.978/ Methylglucamine</td>
<td>2002</td>
<td>Price</td>
<td>No</td>
</tr>
<tr>
<td>C.37.519—Methionine</td>
<td>2002</td>
<td>Price</td>
<td>No</td>
</tr>
<tr>
<td>COMP/E-3/36.700. Industrial and medical gases</td>
<td>2002</td>
<td>Price</td>
<td>No</td>
</tr>
<tr>
<td>COMP/C.37.671—Flood flavor enhancers</td>
<td>2002</td>
<td>Price</td>
<td>No</td>
</tr>
<tr>
<td>COMP/E-2/37.784 FINE ART AUCTION HOUSES</td>
<td>2002</td>
<td>Price</td>
<td>Yes (109,110,111)</td>
</tr>
<tr>
<td>COMP/36.571/D-1: Austrian banks</td>
<td>2002</td>
<td>Price</td>
<td>No</td>
</tr>
<tr>
<td>IV/37.614/F3 PO Belgian beer</td>
<td>2001</td>
<td>Price</td>
<td>No</td>
</tr>
<tr>
<td>COMP/E-1/36.212—Carbonless paper</td>
<td>2001</td>
<td>Price</td>
<td>Yes (233)</td>
</tr>
<tr>
<td>COMP/E-1/36 604—Citric acid</td>
<td>2001</td>
<td>Price</td>
<td>No</td>
</tr>
<tr>
<td>COMP/E-1/36.490—Graphite electrodes</td>
<td>2001</td>
<td>Price</td>
<td>Yes (66)</td>
</tr>
<tr>
<td>COMP/37.800/F3—Luxembourg Brewers</td>
<td>2001</td>
<td>Quantity</td>
<td>No</td>
</tr>
</tbody>
</table>

30 Introducing the possibility for pre-announcements in our model, as in Marshall et al., might help firms coordinate their strategies but would not change the set of sustainable outcomes in collusion with sequential announcements. Indeed, this set cannot be smaller since pre-announcements only expand each firm’s set of actions. And it cannot be larger either, since those pre-announcements that cannot be sustained with announcements would be systematically retracted.

31 In contrast, in the static Stackelberg equilibrium the follower produces less and earns less profit than the leader.
Appendix A (continued)

<table>
<thead>
<tr>
<th>Case</th>
<th>Decision date</th>
<th>Nature of competition</th>
<th>Leadership</th>
</tr>
</thead>
<tbody>
<tr>
<td>COMP/E-1/37.512 Vitamins</td>
<td>2001</td>
<td>Price</td>
<td>Yes</td>
</tr>
<tr>
<td>COMP/E-1/37.027—Zinc phosphate</td>
<td>2001</td>
<td>Price</td>
<td>?(67)</td>
</tr>
<tr>
<td>IV/34.018—Far East Trade Tariff Charges</td>
<td>2000</td>
<td>Quantity</td>
<td>No</td>
</tr>
<tr>
<td>IV/34.250—Europe Asia Trades Agreement</td>
<td>1999</td>
<td>Quantity</td>
<td>No</td>
</tr>
<tr>
<td>IV/34503—Ferry operators</td>
<td>1996</td>
<td>?</td>
<td>No</td>
</tr>
</tbody>
</table>

Appendix B. Proof of Proposition 2

When \( \delta < 1/2 \), price leadership allows the firms to sustain collusion where they could not do so with simultaneous pricing. We now focus on the case \( \delta \geq 1/2 \), in which case some collusion can already be achieved with simultaneous pricing, and denote by \( \Gamma_{\text{sim}} \) the Pareto frontier (from the point of view of the firms) of the set of sustainable collusive outcomes. By construction, any price in \( \Gamma_{\text{sim}} \) belongs to \( [p_i^L, p_i^F] \): if the firms can collude on a price outside this range, replacing this price with \( p^* < p_i^L \), keeping market shares constant, enhances both profits and is also feasible. Therefore, \( \Gamma_{\text{sim}} \) is the set of Pareto outcomes among those that satisfy \( IC_1 \) and \( IC_2 \); it has been characterized by Miklos-Thal (2011) (see Fig. 1, where the curve \( \Gamma \) represents the set of outcomes for which \( IC_1 \) binds while the curve \( \Gamma_{\text{PO}} \) represents the set of the unconstrained Pareto optima, from the point of view of the firms)\(^{32,33} \):

- \( \Gamma_{\text{sim}} \) is empty for any \( \delta < \frac{1}{2} \);
- there exists a threshold \( \tilde{\delta} = \left( \frac{1}{2}, \frac{1}{3} \right) \) such that:
  - for \( \tilde{\delta} = \frac{1}{2} \) (see Fig. 1a), \( \Gamma_{\text{sim}} \) coincides with part of the less efficient firm’s incentive constraint \( IC_2 \) (segment AB);
  - for \( \delta \geq \tilde{\delta} \) (see Fig. 1b), \( \Gamma_{\text{sim}} \) also includes part of the unconstrained Pareto optima (arc AC).

As already noted in the text, whenever a collusive outcome belongs to \( \Gamma_{\text{sim}} \) but not to \( \Gamma_{\text{PO}} \) (that is, any outcome on the vertical frontier AB), relaxing \( IC_2 \) which moves AB to the right, allows for Pareto-improvements; in contrast, relaxing \( IC_1 \) expands the set of Pareto efficient outcomes (by expanding the vertical frontier AB to the top in Fig. 1a, or by expanding the arc AC to the left in Fig. 1b) but cannot Pareto-improve upon the existing Pareto frontier (the additional Pareto-efficient outcomes are less favorable to firm 1). The firms can therefore achieve Pareto improvements if and only if they relax \( IC_2 \).\(^{33} \) We now show that they can indeed do so by setting their prices sequentially, the less efficient firm acting as a leader.

Note first that in any equilibrium \( (p^*, c^*) \) in which firm i acts as a follower, that firm’s incentive constraint \( IC_i \) must hold since it has the same deviation opportunities as with simultaneous pricing (it can replicate any deviation in Stage 2, and cannot do better by deviating in Stage 1). Therefore, the only way to avoid the constraint \( IC_2 \) is to have firm 2 acting as a leader. Conversely, there exists indeed an equilibrium in which firm 2 acts as a leader and \( IC_2 \) need not hold. Consider the following strategies:

- In each period, firm 2 acts as a leader and sets \( p^F \) in Stage 1, while firm 1, acting as a follower, matches this price in Stage 2; market shares are then set to \( c^F = 1 - c^L \);
- Any deviation by either firm is followed by maximal punishments in the following periods; in addition, if in a given period the leader deviates in Stage 1, then in Stage 2 the follower (firm 1) either slightly undercuts it (as long as the leader sets a price above cost in Stage 1) or sets a price equal to its own cost (\( c^L \)).

By construction, firm 2 can never gain from deviating, since any deviation from the leader is immediately punished. Therefore, only the follower’s constraint \( IC_2 \) needs to be satisfied.

Appendix C. Proof of Proposition 3

Consider the following price leadership strategies. In each period, the leader sets a price \( p_i^L \) at Stage 1, while its rival sets a price \( p_i^F \) at Stage 2. Any deviation by either firm is followed by the worst punishment available with simultaneous pricing and, if the leader deviates in Stage 1, then in Stage 2 either the follower best responds to the leader’s price (so as to maximize the profit in the current period) or the two firms play the static Nash equilibrium (if the leader did not set a price in Stage 1).

We now show that, for \( \varepsilon \) small enough, the collusive outcome \( (p_i^L, p_i^F) = (p^* + \varepsilon, p^*) \) is sustainable and more profitable than \( (p^*, p^*) \). We first show that the leader’s maximal deviation profit, \( \pi' \), is lower than the collusive profit \( n(p_i^L, p_i^F) \). Since the leader benefits from an increase in the rival’s price, under our assumptions (strategic complementarity and stability), the Stackelberg equilibrium exhibits higher prices than the static Nash equilibrium, and the leader’s price exceeds the follower’s one. Of the two firms, the follower is thus the one that obtains a larger profit in the Stackelberg outcome, since it faces a higher rival’s price and moreover best responds to it. The condition \( \pi' \geq (\pi - \varepsilon) \) then implies \( \pi' = n(p^*, p^*) > \pi' \) and thus, by continuity,

\[
\pi(p_i^L, p_i^F) = \pi(p^* + \varepsilon, p^*) > \pi' \quad (\pi' < \pi)\]

for \( \varepsilon \) small enough. It is also straightforward to check that the collusive outcome \( (p_i^L, p_i^F) \) increases total profits (that is, \( n(p^* + \varepsilon, p^*) + n(p^*, p^* + \varepsilon) > 2n' \) for \( \varepsilon \) small enough). It remains to show that increasing the price \( p_i^L \) above \( p^* \) relaxes the follower’s no-deviation condition, which writes as:

\[
\max_p n(p, p_i^L) - n(p, p_i^F) \leq \bar{\delta} \left( n(p, p_i^F) - \psi \right)
\]

By assumption, the left-hand side of Eq. (6) equals \( \bar{\delta} \left( n(p, p_i^F) - \psi \right) \), for \( (p, p_i^F) = (p^*, p^*) \) (since then \( n(p, p_i^F) = n' \)); furthermore, increasing \( p_i^L \) by a small amount \( \varepsilon \) increases the follower’s profit \( n(p_i^L, p_i^F) \). In addition, this reduces the short-term gain from a deviation: denoting by \( p_i^L = \arg\max_p n(p, p_i^L) \) the price that best responds to \( p_i^F \) and using the envelope theorem, we have:

\[
\frac{\partial}{\partial p_i^L} \left( \max_p n(p, p_i^L) - n(p, p_i^F) \right) = \partial_1 n(p^*, p_i^L) - \partial_1 n(p_i^L, p_i^F)
\]

where the inequality follows from strategic complementarity (i.e., \( \partial_1 n > 0 \)) and the fact that, since \( p^* \) is by construction strictly higher than the best response to \( p^* \), by continuity \( p_i^L = p^* \) is strictly higher than the best response \( p^* \) to \( p_i^L = p^* + \varepsilon \) for \( \varepsilon \) small enough. The conclusion follows.

Appendix D. Proof of Proposition 4

We focus on the case \( n^2 > n^M \) and suppose that the same firm, acting systematically as a leader, sets its price to \( p_i^L \) in Stage 1 while the
other firm sets its price $p_f$ in Stage 2. The leader then earns $n(p_f, p_f^*)$ while the follower earns $n(p_f^*, p_f^*)$. We look for a collusive outcome $(p_f^*, p_f^*)$ that Pareto dominates the collusive outcome $(p', p')$, that is, satisfying:

$$n(p_f^*, p_f^*) \geq n^R,$$

(7)

$$n(p_f^*, p_f^*) \geq n^F.$$

(8)

with at least one strict inequality. Given Eq. (7) and $n^R \geq n^F$, deviations by the leader are again easy to deter, using either the static Nash or the follower’s best response in Stage 2. If deviations by the follower are punished as with simultaneous pricing, this collusion is sustainable whenever:

$$\max_{p_f} n(p_f, p_f^*) - n(p_f^*, p_f^*) \leq \frac{\delta}{\varepsilon} (n^R - \varepsilon).$$

(9)

Thus, even ignoring the possibility that price leadership may help inflict tougher punishments, a collusive outcome $(p_f^*, p_f^*)$ is sustainable and Pareto-improves upon the best symmetric collusive outcome $(p', p')$ whenever it satisfies Eqs. (7), (8) and (9).

We first note that adopting the Stackelberg outcome, i.e., $(p_f^*, p_f^*) = (p', p')$, strictly satisfies the sustainability condition (9): by construction, the left-hand side is then zero, whereas the right-hand side satisfies

$$\frac{\delta}{\varepsilon} (n^R - \varepsilon) > 0,$$

since $n^R > n^F$ (see Appendix B) and, by construction, $n^R > n^S$. The Stackelberg outcome also strictly satisfies Eqs. (8) for $n^R = n^F$ (since $n^R > n^F$) and weakly satisfies Eq. (7) for $n^R = n^F$. Therefore, by continuity, there exists a neighboring outcome $(p_f^*, p_f^*)$ that strictly satisfies Eq. (9) as well as Eqs. (7) and (8) for $n^R = n^F$. By continuity, there thus exists $n^R > n^F$ such that, whenever $n^R > n^F$, selecting one firm as a systematic price leader allows both firms to obtain more than $n^R$.

To conclude the proof, it suffices to note that both firms cannot increase their profits when $n^R > n^F$.

$$\varepsilon > 0$$

It suffices to choose $(p_f^*, p_f^*) = (p', p' + \varepsilon)$. Since $n(p_f^*, p_f^*) > 0$, Eq. (7) is then strictly satisfied for $\varepsilon > 0$, whereas Eqs. (8) and (9) remain strictly satisfied for $\varepsilon$ small enough.

Appendix E. Proof of Proposition 5

We first show that in any candidate equilibrium in which the firms set their quantities sequentially in a given period, the follower must choose a quantity close to its static best response when $\delta$ goes to zero. Let $q_f$ and $q_f^*(\cdot)$ denote the leader’s quantity and the follower’s strategy in that period (possibly contingent on past history), and $r(q)$ denote the static best response, characterized by

$$\partial_1 n(r(q), q) = 0.$$

(10)

By construction, given the quantity $q$ actually adopted by the leader, the follower must prefer choosing $q_f(q)$ to $r(q)$: since the continuation profit after $q_f(q)$ cannot exceed the monopoly level, $n^M$, while the continuation profit after $r(q)$ cannot be negative, it must therefore be the case that:

$$n(r(q), q) - n(q_f(q), q) \leq \frac{\delta n^M}{1 - \delta},$$

(11)

where:

$$n(q_f(q), q) = n(r(q), q) + \partial_1 n(r(q), q)(q_f(q) - r(q))$$

$$+ \frac{\partial_1 n(q_f(q), q)}{2}(q_f(q) - r(q))^2$$

(12)

for some $q_f(q)$ lying between $q_f(q)$ and $r(q)$. Using Eqs. (10), (11) and (12) imply:

$$(q_f(q) - r(q))^2 \leq \frac{\delta}{1 - \delta} \frac{2n^M}{\partial_1 n(q_f(q), q)} \leq \frac{\delta}{1 - \delta} \frac{2n^M}{B},$$

and thus for any $\varepsilon > 0$ there exists $\eta(\varepsilon) > 0$ such that $\delta < \eta(\varepsilon)$ implies $|q_f(q) - r(q)| < \varepsilon$.

We now show that, as a result, the leader must choose a quantity close to the static Stackelberg in that period. By construction, the leader must prefer the equilibrium quantity $q_f$, followed by a continuation profit that cannot exceed the monopoly level, to the Stackelberg quantity, $q_f^*$, followed by a continuation value that cannot be negative; hence:

$$n(q_f^*(\cdot), q^*) - n(q_f^*(\cdot), q) \leq \frac{\delta n^M}{1 - \delta}.$$

(13)

In case there are several Stackelberg outcomes, we here select arbitrarily one of them.
The regularity conditions yield:

\[ n(q, q_1(q_1)) = n(q, r(q_1)) + \frac{\partial n(q, q_1)}{\partial q_1}|_{q_1 = q_1} \frac{n(q_1(q_1) - r(q_1))}{\partial q_1} \]

where \( q_1 \) (respectively, \( q_2 \)) lies between \( q_1(q_1) \) and \( r(q_1) \) (respectively, between \( q_2(q_2) \) and \( r(q_2) \)). Using these expressions, Eq. (13) implies

\[ n(q^i, r(q^i)) - n(q, r(q_1)) \leq \frac{\partial n}{\partial q} \left| \frac{\partial n(q, q_1)}{\partial q_1} \right|_{q_1 = q_1} |q_1(q_1) - r(q_1)| \]

By construction, the function \( n(q, r(q)) \) is maximal for \( q = q^i \), which implies that the left-hand side must be non-negative. As for the right-hand side, it is lower than:

\[ \frac{\partial n}{\partial q} \left| \frac{\partial n(q, q_1)}{\partial q_1} \right|_{q_1 = q_1} |q_1(q_1) - r(q_1)| \]

Therefore, in the light of the above reasoning, for any \( \lambda > 0 \) there exists \( \mu > 0 \) such that \( \lambda - \mu \) implies:

\[ 0 \leq n(q, r(q_1)) - n(q^i, r(q^i)) \leq \lambda. \]

It follows that the leader's equilibrium quantity \( q_1 \) must be arbitrarily close to the Stackelberg level \( q^i \) for sufficiently low values of \( \lambda \). Since the follower must moreover set a quantity close to its best response, the profits so achieved must therefore be close to the Stackelberg profits, which are here lower than the profit generated by the static Nash equilibrium in simultaneous moves. Indeed, strategic substitutability \( r' < 0 \) implies that the Stackelberg leader's quantity exceeds the simultaneous move Nash quantity \( i.e., q^i > q^{N} \).36 While stability \( r'' > 0 \) implies that total output is higher in the Stackelberg equilibrium than in the simultaneous move Nash equilibrium:

\[ q^i + q^i = q^i + r(q^i) > q^N + r(q^N) = 2q^N. \]

This, in turn, implies that the Stackelberg equilibrium generates less profit than the simultaneous move Nash.39

\[ \text{Appendix F. Proof of Proposition 6} \]

Evaluated at a symmetric outcome \((q, q)\), the leader's no-deviation constraint \((5)\) can be written as:

\[ n(q, q) \geq (1 - \delta)\eta^i + \omega \]

whereas \( \eta \) is defined as the smallest quantity satisfying Eq. (3), and is thus characterized by:

\[ n(q, q) = (1 - \delta)\eta + \omega \]

The assumption \( \partial n/\partial q > 0 \) implies that the Stackelberg leader's quantity increases as \( q \) decreases from the static Nash quantity \( q^N \) to the monopoly level \( q^M \). As noted in the proof of the Proposition 5, \( \max q_n(q, q^*) \) is moreover lower than \( q^N \) for any \( q > q^N \) (since in that case \( \max q_n(q, q^*) = q^N(\delta) \leq q^N \)) under the assumption \( \eta'' > 0 \).\( \max q_n(q, q^*) \) instead exceeds \( q^N \) when \( q > q^M \). There thus exists \( \eta > q^M \), characterized by:

\[ \max q_n(q, \eta) = q^N \]

such that the leader's no-deviation constraint \((14)\) is strictly satisfied for \( q = q^i \) whenever \( q < q^i \), or \( \eta > \eta^i \equiv n(q^i, \eta^i), \) where \( \eta^i < q^M \).

The follower's incentive constraint \((4)\) can be written as:

\[ \Psi(q^i, \eta^i) \geq 0. \]

where:

\[ \Psi(q^i, \eta^i) = \frac{\partial}{\partial q} \left| \frac{\partial n(q, q_1)}{\partial q_1} \right|_{q_1 = q_1} |q_1(q_1) - r(q_1)| \]

By construction, \((q^i, \eta^i)\) satisfies this condition with equality. Consider now a neighboring point of the form \((q^c, \eta^c) = (q^c - \epsilon, q^i + \epsilon)\). We have:

\[ \frac{d}{d \epsilon} \left( \Psi(q^i, \eta^i) \right) |_{\epsilon = 0} = \frac{d}{d \epsilon} \left( \Psi(q^i + \epsilon, \eta^i - \epsilon) \right) |_{\epsilon = 0} = \frac{d}{d \epsilon} \left( q^N + r(q^N) \right) |_{\epsilon = 0} = 0. \]

and thus (using the envelop theorem):

\[ \frac{d}{d \epsilon} \left( \Psi(q^i, \eta^i) \right) |_{\epsilon = 0} = \frac{d}{d \epsilon} \left( \Psi(q^i - \epsilon, q^i + \epsilon) \right) |_{\epsilon = 0} = 0. \]

Using the first-order condition characterizing \( r(q^i) \):

\[ P(\epsilon) + \epsilon r(q^i) + P(\epsilon) - C(\epsilon) = 0. \]

we thus have:

\[ \frac{d}{d \epsilon} \left( \Psi(q^i - \epsilon, q^i + \epsilon) \right) |_{\epsilon = 0} = \frac{d}{d \epsilon} \left( \Psi(q^i + \epsilon, q^i - \epsilon) \right) |_{\epsilon = 0} = 0. \]

It is not possible here to follow the same steps as for price competition (i.e., consider an outcome \((q^c - \epsilon, q^i + \epsilon))\), since a reduction in the leader's quantity exacerbates the short-term gain of a deviation for the follower, instead or reducing it.

To check that \( P(\epsilon) = C(q^i) \), it suffices to note that \( q^c < R(q^i) \), and thus \( P(\epsilon) \cdot C(q^i) = P(\epsilon) \cdot C(q^i) + P(2\epsilon) \cdot q^i > 0 \).
By construction, \( q^* < q^N = r(q^N) \). Taken with the regularity condition \( r'(q) < 0 \), this implies that \( r(q^*) > q^* \). Since \( C(\cdot) \) is increasing while \( P(\cdot) \) is decreasing, we have:

\[
\left. \frac{d}{dq} \Psi(q^* - \epsilon, q^* + \epsilon) \right|_{\epsilon = 0} > 0.
\]

Thus, for \( \epsilon \) positive but small enough:

- the outcome \( (q^* - \epsilon, q^* + \epsilon) \) strictly satisfies the follower's no-deviation constraint (16);
- since the outcome \( (q^*, q^*) \) strictly satisfies the leader's no-deviation constraint (14) when \( r^* = n^{m*} \), the outcome \( (q^* - \epsilon, q^* + \epsilon) \) also strictly satisfies the leader's no-deviation constraint (14);
- and at the first-order, the outcome \( (q^* - \epsilon, q^* + \epsilon) \) generates the same total profit as \( (q^*, q^*) \).

Therefore, by continuity there exists a neighboring outcome \( (\hat{q}_L, \hat{q}_F) \) that satisfies both firms' no-deviation constraints and generates a greater aggregate profit.

**Appendix G. Proof of Proposition 7**

By construction, quantity leadership cannot yield a Pareto improvement when \( r^* = n^{m*} = (d - c)^2/8 \). From now on, we will focus on the case \( r^* < n^{m*} \), which is equivalent to \( q^* < q^M = (d - c)/4 \).

Suppose that in each period the same firm, acting as a leader, sets \( q_f \) in Stage 1 while the follower sets \( q_f \) in Stage 2. The leader thus earns \( \pi(q_f, q_f) \) while the follower earns \( \pi(q_f, q_f) \). We look for a collusive outcome \( (q^*, q^*) \) which is sustainable, that is, satisfies Eqs. (4) and (5), and moreover Pareto dominates the collusive outcome \( (q^*, q^*) \), that is:

\[
\begin{align*}
\pi(q_f, q_f) & \geq \pi^* \\
\pi(q_f, q_f) & > \pi^* 
\end{align*}
\]

with at least one strict inequality.

For the linear case, the set \( P_L \) (resp., \( P_F \)) of outcomes satisfying Eq. (17) (resp., Eq. (18)) can be written as \( q_f \leq \Phi(q_f) \) (resp., \( q_f \leq \Phi(q_f) \)), where \( \Phi(q) \equiv d - c - q - r^* \). By construction, the boundaries of \( P_L \) and \( P_F \), \( \Gamma_L \) and \( \Gamma_F \), intersect at \( (q^*, q^*) \). Moreover, the slope of \( \Gamma_L \) at \( (q^*, q^*) \) is equal to:

\[
\left. \frac{d}{dq} \Phi(q_f) \right|_{q_f = q^*} = 1 - \frac{r^*}{(q_f)^2} = 1 - \frac{d - c - 2q^*}{q_f} = \frac{d - c - 3q^*}{q_f}.
\]

Since \( q^* = \frac{4d - c}{5} \), \( q_f \geq (0, 1) \), \( \Gamma_L \) thus crosses \( \Gamma_F \) from above, as shown in Fig. 2. Moreover, since \( \Phi(q) = -2q^*/q^3 \) for \( q^* > 0 \), the slope of the boundaries is positive in the relevant range. The set \( P \) of outcomes that Pareto-dominate \((q^*, q^*)\) is thus represented by the area, filled with vertical lines in Fig. 2, which lies between (and below) \( \Gamma_L \) and \( \Gamma_F \); in particular, \( P \) lies in the quadrant below \((q^*, q^*)\), that is, it satisfies \( q^* \leq q^* \) (with a strict inequality except for the symmetric outcome \((q^*, q^*)\)).

Let \( \Phi \) denote the set of outcomes satisfying the follower's constraint (4). Since the collusive outcome \((q^*, q^*)\) is just sustainable, \( \Gamma_F \) passes through \((q^*, q^*)\) as well. Furthermore, using \( \Pi(q_f, q_f) = (d - c - q_f - q_f)q_f \) and:

\[
\max \Pi(q(f), q_f) = \left( \frac{d - c - q_f - q_f}{q_f} \right)^2.
\]

the boundary of \( \Phi \), \( \Gamma_F \), is defined by:

\[
\left( \frac{d - c - q_f - q_f}{q_f} \right)^2 = \frac{1}{1 - 1/4} \left( d - c - q_f - q_f \right)q_f - w.
\]

Differentiating this condition yields:

\[
\left[ \left( \frac{d - c - q_f - q_f}{q_f} \right) + \frac{d}{1 - 1/4} \right] dq_f
\]

Therefore, the slope of \( \Gamma_F \) at \((q^*, q^*)\) is equal to:

\[
s_{\phi} \equiv \frac{d}{dq}(q_f = q^*) = -\frac{1}{2} + \frac{\delta q^*}{2 - c - 3q^*}.
\]

In general, the sign of \( s_{\phi} \) is ambiguous, but is negative when \( q^* \) is small (that is, when the punishment is strong); in particular:

**Lemma 1.** If \( q^* < q^M \) and \( w \leq r^* \), then \( s_{\phi} < 0 \).

**Proof.** The slope \( s_{\phi} \) is negative for:

\[
q^* < q^M = \frac{1 - \delta}{1 - 2\delta} (d - c).
\]

Since \( q^* \) is the smallest quantity satisfying Eq. (3) when \( q^* < q^M \), it suffices to show that \( q^* \) strictly satisfies Eq. (3) for \( w = r^* = (d - c)^2 / 16 \) (and thus \( a f t o r i o n \) for \( w < r^* \)); that is, \( q^* \) must satisfy:

\[
\frac{\delta}{1 - 2\delta} \left( (d - c - 2q^*)q^* - \frac{(d - c)^2}{16} \right) - \frac{1}{4} \left( d - c - 3q^* \right)^2 \geq 0,
\]

which, dividing by \( (d - c)^2 \) and replacing \( q^* \) by its expression, amounts to:

\[
\varphi(q^*) = \frac{\delta}{1 - 2\delta} \left( (1 - 2\frac{1 - \delta}{3 - \delta}) - \frac{1}{16} \right) - \frac{1}{4} \left( 1 - 2\frac{1 - \delta}{3 - \delta} \right)^2 \geq 0.
\]

Simple algebra yields:

\[
\varphi(q^*) = \frac{\delta(7 - 10\delta - \delta^2)}{16(1 - \delta)(3 - \delta)^2}.
\]

The conclusion follows from the fact that the polynomial \( 7 - 10\delta - \delta^2 \) is positive in the relevant range in which \( \delta \) is not large enough to allow to sustain \( q^* = q^M = \frac{d - c}{4} \); for \( \delta > 0 \), the polynomial is positive for
\[ \delta \leq \delta = 4\sqrt{2} - 5 = 0.66, \] whereas \( q \) cannot be sustained (even with \( \Psi = 0 \)) whenever \( \delta < 1/3 \). □

By construction, \( (q^*, q^*) \) belongs to \( \Phi \). Furthermore:

**Lemma 2.** \( \Phi \) is convex.

**Proof.** Using Eq. (17), the sustainability condition (4) can be written as:

\[ \Psi(q, q) \geq 0. \]

where:

\[ \Psi(x, y) \equiv \frac{\delta}{1-\delta} \left[ (d-c-x-y)\Psi - \left( \frac{d-c-x}{2} - y \right) \right] ^2. \]

It suffices to show that

\[ \Delta \equiv \Psi(\lambda x_1 + (1-\lambda)x_2, \lambda y_1 + (1-\lambda)y_2) - \lambda (\Psi(x_1, y_1) + (1-\lambda)\Psi(x_2, y_2)) \]

is non-negative for any pair of different outcomes \( (x_1, y_1) \) and \( (x_2, y_2) \) such that \( \Psi(x_1, y_1) = \Psi(x_2, y_2) = 0 \) and any \( \lambda \in [0, 1) \). Routine computations yield:

\[ \Delta = \lambda(1-\lambda) \left[ \left( \frac{d-c-x_1}{2} - y_1 \right) ^2 + \left( \frac{d-c-x_2}{2} - y_2 \right) ^2 \right] \]

\[ + \left[ d-c-x_1 - y_1 \right] ^2 \]

\[ \geq \left( \frac{d-c-x_1}{2} - y_1 \right) ^2 + \left( \frac{d-c-x_2}{2} - y_2 \right) ^2. \]

From \( \Psi(x, y) = 0 \) (which implies \( y > 0 \)) we have, for \( i = 1, 2 \):

\[ \left( \frac{d-c-x_i}{2} - y_i \right) ^2 \]

and thus:

\[ \frac{y_1 y_2}{\lambda(1-\lambda)} = y_1(y_1 - y_2) \left[ \frac{\omega}{1-\delta} + \left( \frac{d-c-x_1}{2} - y_1 \right) ^2 \right] \]

\[ - y_2(y_1 - y_2) \left[ \frac{\omega}{1-\delta} + \left( \frac{d-c-x_2}{2} - y_2 \right) ^2 \right] \]

\[ + y_1 y_2 \left[ \left( \frac{d-c-x_1}{2} - y_1 \right) ^2 + \left( \frac{d-c-x_2}{2} - y_2 \right) ^2 \right] \]

\[ - 2 \left( \frac{d-c-x_1}{2} - y_1 \right) \left( \frac{d-c-x_2}{2} - y_2 \right) \]

\[ = (y_1 - y_2) \left[ \frac{\omega}{1-\delta} + y_1 \left( \frac{d-c-x_1}{2} - y_1 \right) ^2 + y_2 \left( \frac{d-c-x_1}{2} - y_1 \right) ^2 \right] \]

\[ - 2y_1 y_2 \left( \frac{d-c-x_1}{2} - y_1 \right) \left( \frac{d-c-x_1}{2} - y_1 \right) = (y_1 - y_2) \left[ \frac{\omega}{1-\delta} \right] \]

\[ + \left( y_1 \left( \frac{d-c-x_2}{2} - y_2 \right) - y_2 \left( \frac{d-c-x_1}{2} - y_1 \right) \right) ^2 \geq 0. \]

Therefore, \( \Delta > 0 \) if \( y_1 \neq y_2 \); if \( y_1 = y_2 = y \), then \( x_1 \neq x_2 \) and:

\[ \Delta \lambda(1-\lambda) = \left( \frac{d-c-x_1}{2} - y \right) \left[ \left( \frac{d-c-x_1}{2} - y \right) \right] ^2 \]

\[ = \left( x_1 - x_2 \right) ^2 > 0, \]

which completes the proof. □

Therefore, as shown in Fig. 2, \( \Phi \) lies above its tangent at \( (q^*, q^*) \), \( T \), which has a negative slope \( s_p \). Since \( P \) lies in the quadrant below \( (q^*, q^*) \), it follows that \( T \) separates the two sets, which completes the proof of the proposition.

**References**


