Gas Storage Valuation: 
Price Modelling v. Optimization Methods

Petter Bjerksund*, Gunnar Stensland**, and Frank Vagstad***

In the literature, one approach is to analyse gas storage within a simple one-factor price dynamics framework that is solved to optimality. We follow an alternative approach, where the market is represented by a forward curve with daily granularity, the price uncertainty is represented by six factors, and where we impose a simple and intuitive storage strategy.

Based on UK natural gas market price data, we obtain the gas storage value using our approach, and compare with results from a one-factor model as well as with perfect foresight. We find that our approach captures much more of the true flexibility value than the one-factor model.

1. INTRODUCTION

In the literature, the analysis of natural gas storage has traditionally been integrated with the valuation of other activities of the company, for instance production, supply, and demand. However, the existence of a natural gas forward/futures market motivates the use of decision support models from finance. The basic idea is to consider gas storage as a separate asset, and use the market value framework for valuation and utilization of this asset. The company can deal with economic risk by trading in the financial gas market, and cover possible physical imbalances in the spot market.

Several of the methods in recent literature that are applied from finance are aimed at solving the gas storage problem to optimality. One example is Least Squares Monte Carlo (LSMC) of Longstaff and Schwartz (2001), which is applied to gas storage by for instance Boogert and de Jong (2008).

Longstaff and Schwartz (2001) use the LSMC to evaluate an American stock option to optimality, assuming the usual stock price dynamics. However,
the natural gas market consists not only of a spot price but of a whole family of forward prices. Moreover, market data show: (i) clear seasonal price patterns; (ii) considerable price uncertainty; and (iii) more than a few factors are needed to explain the price uncertainty.

In order to solve the gas storage to optimality and still retain computational efficiency, the number of state variables has to be limited. This means that the actual problem has to be simplified substantially. Optimality is at best attained within the simplified framework. So an important question is how much of the “true” gas storage flexibility value is assumed away by reducing the problem to one that can be solved to optimality.

We apply an alternative approach based on a detailed representation of the forward curve and its dynamics, combined with an intuitive and feasible decision rule that follows from repeated maximization of the intrinsic value. At each decision point, the market is represented by an updated forward curve that is consistent with the quoted market prices and that reflects relevant historical information (typical time profiles). We apply Principal Component Analysis (PCA) to identify the (six) factors and determine their loads from actual forward curve movements. We use Monte Carlo simulations to model possible realisations of the forward curve dynamics, where the decision that is locked in at each point in time follows from maximizing the intrinsic value of the gas storage.

Our valuation model is tested on market data from the UK National Balancing Point (NBP) covering a two-year period. We compare the results from our approach with the results from a one-factor optimization model as well as with unfeasible perfect foresight (ex post optimization). The results indicate that our approach, with a rich representation of the forward price dynamics combined with a simple and intuitive decision rule (repeated intrinsic value maximization), captures much more of the actual flexibility value than solving gas storage to optimality within a one-factor model framework.

2. SPOT PRICE MODELS

Boogert and de Jong (2008) present a spot price model for valuation of gas storage. They assume that the spot price dynamics are

\[
\frac{dS(t)}{S(t)} = \kappa [\mu(t) - \ln S(t)] dt + \sigma dW(t)
\]

(1)

where the mean-reversion rate \( \kappa \) and the instantaneous spot volatility \( \sigma \) are positive constants. The long-term level \( \mu(t) \) is a time-varying function that can be used to fit the spot price process to the forward curve at the initial evaluation date. This model is referred to as the log-normal mean-reverting Ornstein-Uhlenbeck price process. Hodges (2004) and Chen and Forsyth (2006) consider a similar price model.

Boogert and de Jong (2008) use (1) above to simulate the spot price paths. These paths, combined with the characteristics of the gas storage, are then
analyzed by LSMC. Chen and Forsyth (2006) combine a regime-switching price model with the storage characteristics to set up an optimal control problem. The solution to this problem gives the storage value.

In our opinion, a one-factor framework is not appropriate for modelling a gas storage. We will give several reasons.

Firstly, the model above is a one-factor spot model. A one-factor model for gas and electricity is unrealistic. Koekebakker and Ollmar (2005) find that up to 10 factors are needed to explain the price movements in the Nord Pool electricity market. Our data indicate that we need six factors to explain 90% of the variation in the NBP gas market in UK.

The reason for choosing a one-factor framework is basically dictated by the focus on the solution method. Boogert and de Jong (2008) find stable results for a number of fewer than 5,000 price paths. Increasing the number of factors will dramatically influence this result.

Suppose we extend the model above to a three-factor spot model. Within the LSMC approach, we then need to estimate a decision rule that is made conditional on the value of these three factors for every time step and for every state of the reservoir. Li (2007) suggests the regression

\[
E_t \left[ \frac{V_{t+1}}{F_t} \right] = \alpha_0 + \alpha_1 \exp(y_1^t) + \alpha_2 \exp(y_2^t) + \alpha_3 \exp(y_3^t)
\]

where \( y_i^t; i = 1, 2, 3 \) are the different factors. The choice might be good but it is still arbitrary. There is an endless number of basis functions of the three factors that are not included above. In this paper we propose a six-factor forward price model. This model is unsuitable for a LSMC solution procedure. The reason is that the estimation of the decision rule will be unfeasible.

Secondly, the initial forward curve used to calculate the trend above plays an unrealistic role. In order to start with a model that is consistent with the current market situation, the trend in the above spot price model is calibrated to the forward market prices that are quoted at the initial date. This creates sensitivity to the initial forward curve that is unrealistic. It will also create problems for a possible hedging strategy.

Thirdly, it is unclear how to back-test the model over a given period. We may run into problems if we want to compare the value calculated initially with the value we obtain from running the storage following this decision rule. In practice we will never assign that much weight on the initial forward curve. Since market prices do not follow a one-factor model we will observe a new trend function every day. Given that we stick with the old trend function, we take positions on a certain market view. The result will depend on whether this market view is profitable or not. The outcome will be very erratic.

And finally, a storage may be interpreted as a complex calendar spread. It is well known from Margrabe (1987) that the flexibility value of an option to
exchange one asset for another is zero if the two asset prices are perfectly cor-
related. One-factor models basically focus on more or less parallel shifts of the
forward curve, which means that forward price returns for different points at the
curve are highly correlated. Such forward curve movements will have limited
impact on the storage decision as well as the value of the storage flexibility.

3. VALUATION MODEL

In the following we describe our method for evaluating the storage. We
adopt the standard assumptions from contingent claims analysis of a complete
market with no frictions and no riskless arbitrage opportunities, see, e.g., Cox
and Ross (1976), Harrison and Kreps (1979), and Harrison and Pliska (1981).

3.1 Forward Curve

We assume that the company faces a decision problem with daily gran-
ularity. Consequently, the company should use information with the same gran-
ularity. The key information in the gas storage problem is the forward curve,
hence we need a forward curve with daily granularity.

It follows from economic decision theory that the current forward price
for a given day may be interpreted as the certainty equivalent value of that day’s
future spot price, given the current information. We argue that the forward curve
should to be consistent with updated market information (quoted forward contract
prices) and relevant historical information (typical time profiles).

3.2 Forward Curve Dynamics

We assume that the risk adjusted dynamics of the forward curve can be
represented by a general multifactor model. See, for instance, Heath, Jarrow, and
Morton (1992), Bjerksund, Rasmussen, and Stensland (2000), or Clewlow and
Strickland (2000). For an overview of different forward curve methods, see Ge-
man (2005).

The dynamics of the forward curve are represented by the following
equation

\[
d\frac{F(t, T)}{F(t, T)} = \sum_{i=1}^{N} \sigma_i(t, T) dZ_i(t)
\]

where \( F(t, T) \) represents the forward price at time \( t \) for delivery at time \( T \). The
volatility is represented by \( \sigma_i(t, T) \) where \( i = 1, \ldots, N \). The increments of the \( N \)
Brownian motions \( dZ_i(t) \) are assumed independent.

The dynamics just above translates into the future forward price being

\[
F(t, T) = F(0, T) \exp \left\{ \sum_{i=1}^{N} \left[ \int_0^t \sigma_i(u, T) dZ_i(u) - \frac{1}{2} \int_0^t \sigma_i^2(u, T) du \right] \right\}.
\]
Every day, we construct a forward curve with daily granularity. Next we find a return function for each day as a function of time to delivery. Then we perform principal component analysis (PCA) to find typical curve movements. The volatility functions follow from the loadings in the PCA.

We assume that the volatility functions are time homogeneous, i.e.,

$$\sigma_i(t,T) = \sigma_i(T-t); i = 1, \ldots, N. \quad (5)$$

Eq. (5) means that the loadings from the PCA depend only on time to delivery (maturity) and not on calendar time (season). The assumption brings our model in line with the one-factor model from the existing literature that we use as our benchmark, where the volatility is not seasonal. However, the absence of seasonality in the factor loadings may be considered a strong assumption. This model is not likely to detect possible seasonal volatility in the data. The assumption might be relaxed if we find ways of estimating PCA-components as a function of calendar time as well as time to delivery.

To perform the simulations we apply the following discrete time representation of (4)

$$F(t + \Delta t, T) = F(t, T) \exp \left\{ \sum_{i=1}^{N} \left( \sigma_i(T-t) \Delta t \bar{\varepsilon}_i - \frac{1}{2} \sigma_i^2(T-t) \Delta t \right) \right\}. \quad (6)$$

Equation (6) is used to simulate the forward curve at time $t + \Delta t$ conditional on the forward curve at time $t$, where we draw $N$ independent standard normal distributed numbers $\bar{\varepsilon}_i, i = 1, \ldots, N$.

3.3 Intrinsic Value

Based on a forward curve with daily granularity we find the optimal deterministic strategy. This is done by solving the following dynamic program where both time and reservoir are discretized

$$V(i,j) = \max_{k \in \{-\min(j, 2), 0, \min(j-1, j-2)\}} \left( e^{-r \Delta t} V(i + 1, j + k) - p(i) u_k - c(i, j, k, p(i)) \right) \quad (7)$$

where

- $V(i,j)$: value function
- $i$: time index (day), $i \in \{1, 2, \ldots, I\}$, where $i \Delta t$: time (years)
- $j$: reservoir index, $j \in \{0, 1, 2, \ldots, J\}$, where $u_j$: reservoir (million therms)
- $k$: storage decision variable, $k \in \{-2, -1, 0, 1\}$,
  - where $-$: deplete $uk$ units of gas
  - $+$: inject $uk$ units of gas
- $p(i)$: daily price from the forward curve
- $c()$: cost function
- $r$ : interest rate per annum
We model a storage technology where the magnitude of the maximum depletion rate is twice the (maximum) injection rate. We assume that at the horizon $I\Delta t$, the storage is returned to its initial size $u_{j_0}$. This is enforced by the terminal condition

$$V(I,j) = \begin{cases} -1000 \text{ million pounds} & \forall j \neq j_0 \\ 0 & j = j_0 \end{cases}$$

The costs may be made dependent on time, reservoir, depletion/injection, as well as the gas price.

Some authors (see, e.g., Eydeland and Wolyniec (2003)) refer to the intrinsic value as the optimal initial plan that might be locked in using the tradable products. The implicit assumption seems to be that the granularity of the decision problem corresponds to the product calendar of the market. Now, suppose a problem with a one-year horizon and that only a one-year contract were traded in the market. The intrinsic value of storage would then be zero, according to the above definition. This is clearly meaningless. In most markets, there are traded forward contracts on a monthly delivery (swaps). The definition above would then disregard the possibility of following one strategy during weekdays and another strategy during weekends, which would translate into an unrealistically low reported number for the intrinsic value.

In the following, we define the intrinsic value as the expected value of storage given the risk adjusted dynamics in Section (3.2) above, conditional on following the best initial deterministic plan. The reason for our definition of the intrinsic value is as follows. A contract on a monthly delivery can be interpreted as a portfolio of contracts on daily deliveries. However, the fact that there is one quoted marked price for the monthly contract does not necessarily mean that each daily contract commands the same (forward) market price. Although daily contracts are not traded except for in the very short end of the curve, we know from empirical data that there typically will be time-dependent price differences in the future, for instance between weekends and weekdays. Since we start out with a forward curve with a higher granularity (daily) than the product calendar in the market, our intrinsic value will exceed the value of the traditional intrinsic plan.

### 3.4 Repeated Intrinsic Value Method

We start out with the initial forward curve and find today’s intrinsic plan. From this plan we lock in the storage decision regarding today. If the decision is to fill, the storage is increased and the cost of filling follows from the current spot price. If the decision is to deplete, the storage reservoir is decreased and the revenue follows from the current spot price. If the decision is to stay put, there is no cash flow and the reservoir is unaltered.

Next we simulate a realisation for tomorrow’s forward curve conditional on today’s forward curve and the price dynamics given in Section (3.2). For this
new forward curve, tomorrow’s intrinsic plan is determined. This problem is
updated for the storage decision that is already locked in as well as the passage
of time (one day closer to the terminal date). This plan is used to lock in the
storage decision regarding tomorrow. We continue this procedure until we reach
the terminal date of our problem.

This procedure gives us one possible realisation of daily cash flows from
the storage over the relevant time horizon. We repeat the procedure above to
obtain the desired number of possible cash flow realisations. The value of storage
is given by the average net present value of the simulated daily cash flows.

3.5 Comparing with a One-factor Optimization Model

In this paper we claim that the focus on stochastic optimization methods
leads to an oversimplification of the price modelling that severely undervalues
the value of storage. In order to compare results, we consider the one-factor model

The starting point is the discrete-time version of (1), where the spot price
dynamics is calibrated to the initial forward curve by

\[
S(t + \Delta t) = F(0,t + \Delta t)e^{\ln \left( \frac{S(t)}{F(0,t)} \right) + \frac{1}{2} \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa t})}
\]

\[
-\frac{1}{2} \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa(t + \Delta t)}) + \sqrt{\frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa\Delta t})}\tilde{\varepsilon}
\]

(8)

where and are the spot prices at time and , and
are the initial forward prices for delivery at time and , respec-
tively, \( \tilde{\varepsilon} \) is standard normal, and \( \Delta t \) is the time step size. We use (8) to simulate
possible spot price path realisations. Each path is mapped into a discrete spot
price state space (51 states). Based on the discrete spot price path realisations, we
obtain a spot price transition matrix for each calendar day. Within this framework,
the storage problem is solved to optimality by stochastic dynamic programming.

4. STORAGE VALUATION—AN EXAMPLE

4.1 Data

We use natural gas price data quoted at NBP from October 1st 2004–September 30th 2006. The prices are quoted by pence/therm. The traded contracts
are spot, day ahead, balance of week, balance of month, as well as calendar
months and seasons (summer and winter).

We use the Elviz Front Manager software to translate this price information into a forward curve with daily granularity that is consistent with the
market prices of the quoted contracts and that reflects typical seasonalities over
the week and over the year. A detailed outline, which includes a solution method, is given in Benth, Koekebakker and Ollmar (2007).

The forward curve at NBP at October 1st 2004 is illustrated in Figure 1. Observe that there are price variations within the week (lower prices during weekends) and within the year (higher prices during the winter season).

**Figure 1: Forward Curve on Oct 1st 2004**

![Figure 1: Forward Curve on Oct 1st 2004](image)

Principal components analysis (PCA) is a widely used method for simplifying complex data structures. The basic idea is to identify the most important factors (principal components) and use them to represent the market. For an introduction to PCA of forward curves, see for instance Blanco et al. (2002). Based on the forward curve movements the first year, we apply PCA to estimate the loadings. The loadings of the first six components are given in Figure 2.

**Figure 2: Loadings of the Six Most Important Factors**

![Figure 2: Loadings of the Six Most Important Factors](image)
The first component can be recognized as a parallel shift of the forward curve, which typically will have a small impact on the gas storage value. The second component seems to represent a tilting of the forward curve, but it is far from a linear tilt. The third component can be interpreted as a change in curvature. The other three components are more difficult to give intuitive explanations. However, the combinations of these six factors give rise to a rich class of possible forward curve movements.

The explanation power of the components is shown in Figure 3. Observe that the first factor (parallel shift) explains about 35% of the variation, whereas 75% of the variation can be attributed to the three first factors. We want to explain about 90% of the price variation, hence we settle for six factors.¹

**Figure 3: Cumulative Percentage Variance Contribution of Principal Components**

In Figure 4 we present the volatility term structure for the whole dataset. The graph using only six factors gives a similar picture.

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1. The eigenvalues associated with the 6 first components are 0.27; 0.17; 0.12; 0.05; 0.04; and 0.02.
We would expect to find a falling volatility curve in the natural gas market. The reason is that new information influences the short end of the curve more than the more distant contracts. This is not always the case when inspecting empirical data.

We observe a bump in the volatility around 30 days to delivery. This unexpected effect might follow from the procedure used to create the forward curves. In the short end of the curve the following contracts exist: Next day, next weekend, and next week (weekdays), in addition to balance of month (BOM, i.e., the rest of the month from tomorrow). We use close prices. We know from other markets that these close prices might involve arbitrage. At the Nord Pool power market, for instance, the close price for Quarter1 does not exactly sum up to the weighted average of the close prices for January, February, and March (even if we take into account the time value of money). Since nobody can trade on the close, but rather on the previous bid ask spread, it is not possible to utilize this in trading. Consider an example where we have a price for delivery from $t_0$ to $t_1$ and a price for delivery from $t_0$ to $t_2$. This gives a computed price for $t_1$ to $t_2$. Some mismatch between the close on the price for $t_0$ to $t_1$ and the price for $t_0$ to $t_2$ will be visible in the price for $t_1$ to $t_2$ which in this case is a kind of residual. In addition this residual is glued together with the next month. In some situation the smoothest curve gives the lowest price within the BOM for the last day of the month. Changes of this last day of the month and the beginning of the next month are influenced by changes in this residual. Some of the change in the residual is noise created by our procedure.
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4.2 Storage Characteristics

The characteristics of the stylized storage are given in Table 1.

**Table 1: Storage Characteristics**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>125 million therms</td>
</tr>
<tr>
<td>Terminal</td>
<td>125 million therms</td>
</tr>
<tr>
<td>Max</td>
<td>250 million therms</td>
</tr>
<tr>
<td>Injection</td>
<td>2.5 million therms per day</td>
</tr>
<tr>
<td>Depletion</td>
<td>2.5 or 5 million therms per day</td>
</tr>
</tbody>
</table>

Hence, an empty storage can be filled in 100 days, whereas a full storage can be emptied in 50 days. All costs are set to zero, and we disregard the positive interest rate. We consider a problem with a one-year horizon, and our objective is to obtain the value of the cash flow from operating the storage.

4.3 In-sample Analysis

We start with an in-sample valuation example, where we consider the value of operating the storage facility the first year (October 1st 2004 to September 30th 2005), given the initial forward curve (Figure 1) and using the PCA factor load estimates from the same period (Figure 2).

The traditional intrinsic value that can be locked in using the traded monthly products is 58 million pounds. The intrinsic value using daily resolution of both the forward curve and the storage strategy is 77 million pounds. The one-factor model in Equation (8) (5 000 simulations used to generate transition matrices), optimizing the storage strategy, gives a value of 104 million pounds. Here we have used the short term volatility of 149% per annum and a mean-reverting rate of 0.05 per day, which translates into a rate of $\kappa = 0.05 \cdot \frac{365}{H}$ per year. The short term volatility is a proxy for the short term volatility in the PCA-components, whereas the mean-reverting rate is equal to the rate used by Boogert and de Jong (2008).

Given the initial forward curve and the estimated factor loadings, the six-factor model with repeated intrinsic value maximization gives a storage value of 187 million pounds (1 000 simulations). The standard deviation of this estimate is 1.1 million pounds. This means that the value of flexibility is considerably higher than reported by the other models. In order to obtain rather low standard deviation in the estimate using only 1 000 simulations, we use the value of the financial hedging strategy as a control variate (see Section 5).2

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2. The mean and standard deviation over all simulations for the profit-and-loss from the physical ( unhedged) storage is 190 and 78 million pounds, respectively. This gives a standard deviation of the
As a comparison, we find that the optimal but unfeasible perfect foresight method gives a storage value 205 million pounds (1 000 simulations). The standard deviation of this estimate is 2 million pounds. This result is obtained by simulating all the forward prices paths to the horizon, finding the corresponding spot price path, and the ex post best strategy for each path.

Our results are summarized in Table 2.

### Table 2: Calculated Storage Value 2004 (million pounds)

<table>
<thead>
<tr>
<th>Method</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intrinsic—monthly granularity</td>
<td>58</td>
</tr>
<tr>
<td>Intrinsic—daily granularity</td>
<td>77</td>
</tr>
<tr>
<td>One-factor model</td>
<td>104</td>
</tr>
<tr>
<td>Dynamic intrinsic method</td>
<td>187</td>
</tr>
<tr>
<td>Perfect foresight</td>
<td>205</td>
</tr>
</tbody>
</table>

On this background we claim that the simple repeated intrinsic method is close to the optimal value. We know that the perfect foresight value of 205 million pounds is impossible to obtain.

### 4.4 Out-of-sample Analysis

Next we consider an out-of-sample analysis. In this example, we consider the value of operating the storage facility the second year (October 1st 2005 to September 30th 2006), using the initial forward curve at October 1st 2005 and the PCA factor load estimates from the first year (October 1st 2004 to September 30th 2005).

**Figure 5: Forward Curve Comparison**

The physical storage value estimate (1 000 simulations) of 2.5 million pounds. The mean and standard deviation over all simulations for the profit-and-loss from the financial hedge is –3.4 and 83 million pounds, respectively. This gives a standard deviation of the financial hedge value estimate (1 000 simulations) of 2.6 million pounds. The correlation between the profit-and-loss from the physical storage and the financial hedge is –0.90.
In this case, the traditional intrinsic value that can be locked in using the traded monthly products is 61 million pounds, whereas the intrinsic value of the storage with daily resolution is 74 million pounds. These numbers are quite close to the corresponding values of the first year reported in Table 2 above. To explain this, observe that the storage may be interpreted as a complex calendar spread, and that forward curves at October 1st 2004 and October 1st 2005 basically have the same shape (relative prices), c.f. Figure 5.

The one-factor model, where we use Equation (8) (5,000 simulations) to generate transition matrices, and assume the same short-term volatility and mean reverting rate as above, gives a value of 116 million pounds. The repeated intrinsic method gives a value of 228 million pounds (1,000 simulations using control variate). The standard deviation of this estimate is 1.5 million pounds. As a comparison, we find that the optimal but infeasible perfect foresight model gives 258 million pounds (1,000 simulations). The standard deviation of this estimate is 3 million pounds.

Our findings are summarized in Table 3.

<table>
<thead>
<tr>
<th>Table 3: Calculated Storage Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005 (million pounds)</td>
</tr>
<tr>
<td>-----------------------------------</td>
</tr>
<tr>
<td>Intrinsic—monthly granularity</td>
</tr>
<tr>
<td>Intrinsic—daily granularity</td>
</tr>
<tr>
<td>One-factor model</td>
</tr>
<tr>
<td>Dynamic intrinsic method</td>
</tr>
<tr>
<td>Perfect foresight</td>
</tr>
</tbody>
</table>

4.5 Conclusions

The two examples show that our six-factor model, combined with repeated intrinsic value maximization, creates a significantly higher value than the one-factor stochastic optimization model. Moreover, we find that there is a marginal additional value from perfect foresight. Perfect foresight is of course unfeasible, and represents an upper bound to the true storage value. This indicates that our valuation approach captures most of the storage flexibility value, and suggests that there is a limited potential for improvement of our approach in this

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3. The mean and standard deviation over all simulations for the profit-and-loss from the physical (unhedged) storage is 233 and 98 million pounds, respectively. This gives a standard deviation of the physical storage value estimate (1 000 simulations) of 3.1 million pounds. The mean and standard deviation over all simulations for the profit-and-loss from the financial hedge is –4.6 and 105 million pounds, respectively. This gives a standard deviation of the financial hedge value estimate (1 000 simulations) of 3.3 million pounds. The correlation between the profit-and-loss from the physical storage and the financial hedge is –0.88.
example. Of course, there might be other storage characteristics not investigated here where one-factor models are more suited.

In our examples, we have abstracted from injection/depletion costs. If such costs are included, the investigated methods will give a lower storage value. However, it might be that the that dynamic intrinsic value method, which in a sense utilizes more of the changes in curvature, will suffer a greater reduction in value as compared to the one-factor model.

5. FINANCIAL HEDGE

A crucial point in option pricing theory is the concept of a replicating portfolio. The rationale behind the Black-Scholes option pricing formula is the existence of a dynamic self-financing trading strategy that gives exactly the same pay-off at the horizon. To rule out arbitrage, the option price must coincide with the cost of creating the replicating portfolio. If we know how to replicate an option, we also know how to hedge it, because the two strategies are opposites. However, the beauty of the theory often breaks down in practice. The most important explanation is a mismatch of volatilities between the model and the market.

In the case of risk management of a gas storage, it may be instructive to think in terms of a replicating portfolio. It will in typically consist of positions in all of the forward contracts. Since we are not even close to a formula for the gas storage value, the load on every contract has to be simulated.

Consequently, we use an alternative approach to the hedging problem. In particular, we will exploit our dynamic intrinsic plan to define a financial forward trading strategy. The cash flow from this strategy will be highly negatively correlated with the cash flow generated from the gas storage. However, the strategy will not capture the pure flexibility value that accrues to the owner of the physical facility. Hence we might call this a sub-replicating hedge portfolio.

As an illustration, consider October 1st 2004. Suppose that we can trade daily forward contracts at each point in time. The initial hedge portfolio would then have an exposure as shown in Figure 6.
Suppose instead that only monthly contracts are traded. By mapping the daily exposures from the intrinsic storage plan to net monthly exposures, we would have an exposure at October 1st 2004 as shown in Figure 7.

We consider a dynamic situation where we utilize the storage according to the storage strategy that follows from repeated intrinsic value maximization. In addition, we follow a dynamic financial hedge strategy. Each day, we liquidate yesterday’s hedge portfolio and take new positions that are opposite of the updated intrinsic value maximization storage plan.
Recall that the hedge portfolio consists of forward contract positions. This means that the market value of entering each position (as well as the portfolio) is zero. Next day, new forward prices are quoted. The old hedge portfolio is then liquidated, yielding a positive or negative cash flow. Thereafter, the new hedge portfolio is composed using information from the updated intrinsic value storage plan. And so forth. It follows from above that the market value of launching the dynamic hedge strategy is zero.

Observe that the hedge strategy can be implemented by any market participant. The gas storage ownership per se is no prerequisite for trading in the financial gas market. The storage can of course serve as collateral for the company’s financial gas trading and speculation, but that is a different matter.

So why bother with such a strategy? The explanation is that the cash flows from the gas storage and the hedge portfolio are highly negatively correlated. One obvious practical application of our hedge strategy is to reduce the risk from owning and utilizing a gas storage. Another application is to improve the accuracy of our gas storage value estimate by using the financial hedge strategy as a control variate.4

6. BACK-TEST

In this section, we investigate the performance of our dynamic intrinsic value maximization strategy on the spot prices that actually were realized in the market. We assume the same storage characteristics as above and that terminal storage equals the initial storage.

6.1 Year 1—Storage

We start at October 1st 2004 and consider the following year. Maximizing the intrinsic value that day gives the strategy that is illustrated in Figure 8.
The expected cumulative cash flow from sticking to this strategy is illustrated in the figure below, where we use the property that each day’s forward price can be interpreted as the expected future spot price that day (using the equivalent martingale measure).

The cumulative expected cash flow starts out negative, which reflects that the strategy is to start injecting. The cumulative expected cash flow at the horizon is the initial intrinsic value of 77 million pounds, c.f. Table 2.

We perform an intrinsic value model every day given updated information. In this example, we decide to fill the storage with maximum capacity.
The forward curve given in Figure 10.

**Figure 10: Forward Curve October 2nd 2004**

We now optimize the intrinsic value for the remaining period given updated information. The current reservoir is 127.5 million therms. There are 364 days to the horizon, and the terminal storage must equal 125 million therms.

Note that the strategy is almost unchanged. The best decision day two given that we follow the updated intrinsic plan is also to inject. We continue in this way for every day. For the weekend we use the curve given on last Friday to take the decision whether to inject or deplete.

We continue in this way until September 30th 2005. At this time the reservoir equals 125 million therms. The result for the dynamic intrinsic method from October 1st 2004 until September 30th 2005 is given in the following figures.

The evolution of the storage is given in Figure 11.
Figure 11: Actual Storage Size with the Dynamic Intrinsic Method Oct 1st 2004 to Sept 30th 2005

The spot prices that were realized the first year is shown in Figure 12.

Figure 12: NBP Prices Oct 1st 2004 – Sept 30th 2005

The development of the accumulated cash flow from following the dynamic intrinsic method is shown in Figure 13.
The result from the dynamic intrinsic value method is 48 million pounds. As a comparison, the result from perfect foresight (ex post optimization using the realized spot prices in Figure 12) is 60 million pounds.

6.2 Year 1—Hedged Storage

Above, we found that the realised cash flow from storage by implementing the repeated intrinsic value maximization from 1st October 2004 to 30th September 2005 is 48 million pounds.

Now, suppose that the owner of the gas storage had launched a dynamic financial hedge strategy as described in the previous section. Assuming a market with daily forward contracts, the realised cash flow generated by the dynamic financial hedge portfolio was 185 million pounds. This large number can be explained by the realized seasonal spreads are reduced as compared to the initial spreads. This adds up to a total cash flow from the hedged storage 233 million pounds. The development of the cumulated cash is illustrated in Figure 14.
It may be argued that daily forwards are unrealistic. In order to model a more realistic trading strategy, we assume that all exposure from the optimal intrinsic plan is mapped into net positions in the traded products. For simplicity we assume that the next 10 days are tradable, and thereafter only the relevant trading list from next month and to the horizon. The cash flow generated by the dynamic financial hedge is then reduced to 89 million pounds. This translates into a total cash flow from the hedged storage of 137 million pounds. The results of our back-test are summarized in Table 4.

Table 4: Back-test Results 2004 (million pounds)

<table>
<thead>
<tr>
<th>Storage strategy</th>
<th>Physical storage</th>
<th>Financial hedge</th>
<th>Hedged storage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perfect foresight (unfeasible)</td>
<td>60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dynamic intrinsic method</td>
<td>48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hedge strategy</td>
<td></td>
<td>89</td>
<td>137 (= 48 + 89)</td>
</tr>
<tr>
<td>Trading calendar</td>
<td>89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Daily forwards</td>
<td>185</td>
<td></td>
<td>233 (= 48 + 185)</td>
</tr>
</tbody>
</table>

6.3 Year 2—Storage

Now we perform the same analysis for the year starting at October 1st 2005 with a one-year horizon. The reservoir is the same as in the previous example 125 million therms. The first day we obtain the following intrinsic value results.
The expected cumulative cash flow from sticking to the initial intrinsic plan next year is 74 million pounds. This is close the initial guess the year before. It is the seasonal spread in the forward curve that creates this value. It is therefore fair to say that this pattern is unchanged.

Next we perform a new intrinsic value model every day until September 30th 2006. The reservoir is left at the same size as we started with, that is 125 million terms. The result from this strategy is presented below. The evolution of the storage is given in Figure 16.

The spot prices that were realized the second year are shown in Figure 17.
The development of the accumulated cash flow from following the dynamic intrinsic method is shown in Figure 18.

The result from the dynamic intrinsic value method is 115 million pounds. As a comparison, the result from perfect foresight (ex post optimization using the realized spot prices in Figure 17) is 199 million pounds.
6.4 Year 2—Hedged Storage

Assume a market with daily forward contracts. The dynamic hedging strategy in this case gives a trading profit of 325 million pounds. So the value of the hedged storage amounts to 440 million pounds.

If we only include the 10 first days of forward prices and the trading calendar which starts with the next full month, the result drops to 190 million pounds. The results of our back-test are summarized in Table 5.

Table 5: Back-test Results 2005 (million pounds)

<table>
<thead>
<tr>
<th>Storage strategy</th>
<th>Physical storage</th>
</tr>
</thead>
<tbody>
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<td>Perfect foresight (unfeasible)</td>
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<tr>
<td>Dynamic intrinsic method</td>
<td>115</td>
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<tr>
<td>Hedge strategy</td>
<td>Financial hedge</td>
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<td>Trading calendar</td>
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<tr>
<td>Daily forwards</td>
<td>325</td>
</tr>
</tbody>
</table>

7. CONCLUSION

One approach in the literature is to evaluate a storage by models using a simple price process and complicated optimization methods. We suggest an alternative approach, with a rich representation of prices (forward curve with daily resolution) and uncertainty (six factors), combined with a simple intuitive decision rule (repeated maximization of intrinsic value).

We consider a situation with a one-year horizon and daily storage decisions. The storage technology is such that an empty storage can be filled in 100 days, whereas a full storage can be emptied in 50 days. We abstract from storage costs. Based on market price data from the UK gas market, we compare the results from our model with a one-factor optimization model from the literature and with the unfeasible perfect foresight model. We find that our model captures much more of the true flexibility value than the one-factor model optimization model.

Our results indicate that a multi-factor model combined with a simple intuitive decision rule is a more appropriate framework for analyzing complex assets like gas storage than a one-factor model that is solved to optimality. This suggests that modelling of prices and price dynamics (e.g., seasonal volatility) is a promising area for further research.

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