Volatility Dynamics and Seasonality in Energy Prices: Implications for Crack-Spread Price Risk

Hiroaki Suenaga* and Aaron Smith**

We examine the volatility dynamics of three major petroleum commodities traded on the NYMEX: crude oil, unleaded gasoline, and heating oil. Using the partially overlapping time-series (POTS) framework of Smith (2005), we model jointly all futures contracts with delivery dates up to a year into the future and extract information from these prices about the persistence of market shocks. The model depicts highly nonlinear volatility dynamics that are consistent with the observed seasonality in demand and storage of the three commodities. Specifically, volatility of the three commodity prices exhibits time-to-delivery effects and substantial seasonality, yet their patterns vary systematically by contract delivery month. The conditional variance and correlation across the three commodities also vary over time. High price volatility of near-delivery contracts and their low correlation with concurrently traded distant contracts imply high short-horizon price risk for an unhedged position in the calendar or crack spread. Price risk at the one-year horizon is much lower than short-horizon risk in all seasons and for all positions, but it is still substantial in magnitude for crack-spread positions. Crack-spread hedgers ignore nearby high-season price risk at their peril, but they would also be remiss to ignore the long horizon.

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1. INTRODUCTION

Demand for motor gasoline in the United States peaks in the summer driving season, whereas demand for heating oil peaks in winter. Because these two refined petroleum products are imperfect substitutes in the production pro-
cess, their mismatched seasonal pattern leads oil refiners to produce more gasoline than is needed in winter and more heating oil than is needed in the summer. Inventories therefore accumulate in the winter for motor gasoline and in the summer for heating oil. This inherent seasonality, combined with storability of the commodities, induces nonlinearities in the price dynamics and causes the price risk exposure of refiners and other market participants to vary not only by season, but also by planning horizon in a way that interacts with season. For example, price risk exposure at the six month horizon differs depending on whether that horizon spans an inventory accumulation phase or a peak-demand season. In this article, we quantify how price risk exposure varies by season and planning horizon.

The most volatile seasons and planning horizons exhibit the greatest price risk exposure and therefore provide the greatest potential variance reduction from hedging. However, they also provide the highest probability of default on a hedge position. Default can occur if prices change by more than the margin amount held by the counterparty or clearing house. Counterparties and clearing houses could reduce their exposure by imposing higher margin requirements on positions that cover the most volatile seasons and planning horizons. These considerations may become even more important in the coming years as new financial legislation\(^1\) in the United States appears destined to raise the cost of hedging and push more hedging transactions into central clearing houses.

Futures markets play an important role in risk management, not only by providing low-cost instruments with which to hedge, but also by generating signals about likely future prices. On organized exchanges, multiple futures contracts trade simultaneously, each with a different delivery date. The set of futures prices observed on a particular day reveals market expectations about price dynamics in the ensuing months. For the two refined commodities, this constellation of futures prices exhibits pronounced nonlinearities due to seasonal demand cycles and storability. As implied by the theory of storage (e.g., Williams and Wright, 1991), near-to-delivery contracts are linked by temporal arbitrage to far-from-delivery contracts during inventory accumulation periods. In contrast, a tight supply-demand balance induces high price volatility in peak-demand seasons, which often cannot be mitigated through storage. In this event, the temporal arbitrage link weakens and futures prices for delivery after the peak season will be unaffected by current price shocks. These price patterns for the two refined commodities spill over each other and to crude oil markets via their links through the refinery process.

In this paper, we model at the daily frequency the volatility dynamics of the three major petroleum commodities traded on the NYMEX: crude oil, unleaded gasoline, and heating oil. Using the partially overlapping time-series (POTS) framework of Smith (2005, see also Suenaga, Smith and Williams, 2008), we model jointly all contracts with delivery dates up to a year into the future and

\(^1\) Dodd-Frank Wall Street Reform and Consumer Protection Act, 2010.
extract the information available from the markets about the persistence of price shocks across the three commodities. We uncover highly nonlinear volatility dynamics that are consistent with the observed seasonality in demand and storage. Specifically, we quantify the high price volatility of near-delivery contracts for all three commodities, which exhibit weak correlation with the concurrently traded more distant contracts. This volatility of near-delivery contracts is particularly high during the peak-demand seasons for the two refined commodities, which translates into significant short-horizon price risk for crack- and calendar-spread positions in the peak-demand seasons. In all seasons and for all positions, price risk at the one-year horizon is much lower than short-horizon risk, but it is still substantial in magnitude even for crack-spread positions. At the one-year horizon, crack-spread volatility remains 30 percent of its level for the individual commodities, which implies significant variance reduction from crack-spread hedging at this horizon.²

Numerous recent studies have examined the co-variability of crude oil and refined petroleum product prices (Asche et al., 2003; Girma and Paulson, 1999; Haigh and Holt, 2002; Lanza et al., 2005; Ng and Pirrong, 1996; Serletis, 1994). In contrast to our approach, these studies commonly analyze a single price series per commodity, thereby discarding much of the information available from the markets about the persistence of price shocks. These studies employ the cointegration method, which is useful for depicting the long-run equilibrium relationship across price levels and for showing how each series responds on average to deviations from the equilibrium, but it neglects the price dynamics induced by seasonality in demand and storage. In particular, an application of the standard cointegration approach presumes that the cointegration vector is time-invariant even though demand seasonality differences across commodities imply a seasonal equilibrium price relationship. Furthermore, the standard error correction model presumes the stochastic dynamics of the short-term deviation to be time-invariant.³ In contrast, the theory of storage implies that futures prices respond differently to current market shocks depending on physical inventory, which exhibits substantial seasonal variation both systematically and non-systematically, especially for heating oil and unleaded gasoline.

The rest of the paper is organized as follows. Section 2 reviews the peculiarities of three petroleum commodity markets in the United States and identifies key features expected for volatility and covariability of their prices. Section 3 develops the model that is capable of capturing these features. Section 4 de-
scribes the data and presents the estimation results with emphasis on their implications for the volatility dynamics of three commodities and three widely considered crack-spread positions. Section 5 concludes the paper.

2. SEASONALITY IN PETROLEUM PRODUCT DEMAND AND ITS IMPLICATIONS FOR VOLATILITY DYNAMICS

Three market features cause petroleum commodity prices to exhibit complex stochastic dynamics. First, demand for motor gasoline and heating oil in the United States exhibits strong seasonality with the former peaking in the summer driving season and the latter in winter for space heating. Second, finished petroleum products, manufactured from crude oil and other hydrocarbons, are imperfect substitutes in the refining process. A conventional refining process heats crude oil in a distillation column and extracts motor gasoline, distillate fuel oil, and other refined petroleum products at different temperatures. Since the early 1980’s, refineries have expanded the capacity for secondary refining to reprocess the heavier fractions of refined products into more valuable, lighter products. However, such an alteration of the production mix incurs additional cost. Besides, the capacity of coker or downstream processing cannot be expanded in short run, providing refiners with only a limited flexibility in the proportion of the finished products extracted from a unit of crude oil.

Third, environmental regulations require the specifications of motor gasoline to be altered across seasons (EIA, 1997). In particular, NYMEX defines three different commodity specifications; the Summer grade (delivered from April to mid September), the Winter grade (November to the end of February), and the intermediate specification (delivered in the two transition periods). The shift in product specification requires refiners to clear inventory and adjust their production schedules prior to the change in the product specification. Together with the limited substitutability in the refining process, this regulatory requirement constrains refiners’ production and storage decisions.

In Figure 1, the supply of heating oil and motor gasoline exhibits seasonal variation that is similar in pattern but much smaller than the seasonal variation in their respective demands. For both commodities, monthly consumption generally exceeds net production in their peak demand seasons, of which the difference is supplemented by releasing inventory. For heating oil, this is seen in a rapid decline of inventory from December to March. Inventory reaches its lowest point around the end of winter. It then gradually increases from March to August and stays around the same level until it is depleted during the subsequent winter.

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4. Figure 1 illustrates monthly averages of demand (product supplied for end use and for refinery and blender plus exports), supply (U.S. domestic production plus imports), and inventory for each of the three commodities, over the period between 1981 and 2008. For heating oil, the figure plots the monthly averages of demand, supply, and storage of distillate fuel oil.
Figure 1: Seasonality in Demand, Supply, and Storage of U.S. Crude Oil, Unleaded Gasoline, and High Sulfur Distillate Fuel Oil

(a) Monthly averages of consumption, net production, and storage of crude oil

(b) Monthly averages of consumption, net production, and stock of distillate fuel oil

(c) Monthly averages of consumption, net production, and stock of finished motor gasoline

Note: Three figures are created based on the EIA’s data on the U.S. supply and disposition of crude oil and finished petroleum products (obtained from http://tonto.eia.doe.gov/dnav/pet/pet_sum_snd_d_nus_mbbld_m_cur.htm) for the period between Jan. 1981 and Dec. 2008. For CO, net production represents the sum of the U.S. field production, import, and supply adjustment whereas consumption represents the sum of the U.S. refinery and blender net input, export, and product supplied of crude oil. For HO and UG, net production represents the sum of the U.S. refinery and blender net production, import, and supply adjustment whereas consumption represents the sum of the export and product supplied of each petroleum commodity.
For motor gasoline, inventory is the lowest in August right after the summer driving season. It is also low in October when the commodity specification shifts from the summer to winter grade. Inventory starts building up in November and increases until it reaches an annual high typically in January. During this post-summer season, gasoline production is kept high because a large amount of heating oil needs to be co-produced and stored to meet high winter demand. High gasoline inventory accumulated in this period is released during the subsequent winter before the specification switches to the summer grade in mid March. Since demand is low throughout the winter, contemporaneous supply is kept low and many refineries perform regular maintenance in this period. The production level starts increasing in April and peaks in July. Over the same period, the inventory increases initially and then quickly depletes to meet high demand over the summer driving season.

Whereas the seasonal demand patterns observed for the two refined products imply higher average prices in summer for unleaded gasoline and in winter for heating oil, inter-temporal price differentials are smoothed out through storage, albeit imperfectly. According to the theory of storage (e.g. Williams and Wright, 1991), the equilibrium constellation of spot and futures prices represents the prices at which the marginal benefit of current consumption is equal to or above the expected marginal benefit of future consumption plus the cost of carry. The weak inequality stems from the constraint that firms cannot borrow inventory from the future when the market experiences a shortage of the physical commodity. This constraint creates a discontinuity in the inter-temporal price linkage; two prices are linked by temporal arbitrage when inventory is plentiful whereas they are disconnected when discretionary inventory is zero. In practice, transaction costs (Brennan, Williams, and Wright, 1997) and convenience yield (Gibson and Schwartz, 1990) act to smooth this discontinuity by preventing inventories from becoming zero. Convenience yield reflects the flexibility and reduced costs associated with having inventories on hand, and it tends to be largest at low inventory levels. When convenience yield is high, speculative storage plays a smaller role in determining prices and the inter-temporal price linkage weakens.

The data in Figure 1 indicate that average inventory ranges from 20–24 days of supply for gasoline and 31–37 days of supply for heating oil, depending on the season. Average crude oil inventories range from 23–24 days of supply. The span of these ranges reflects the seasonality in demand and storage costs for each commodity. Overall, the number of days supply in inventory is low for the energy complex compared to many other commodities. For example, U.S. corn inventories average about three months of supply and gold inventories average much more than a year of supply. Low average inventory levels suggest a somewhat limited ability of above-ground storage to smooth price shocks, even in seasons with relatively high inventory. Thus, shocks to current supply and demand will not transmit equally to futures prices with distant delivery dates. These shocks will affect prices with nearby delivery dates more than prices with distant delivery dates, thereby inducing some mean reversion in spot prices and implying greater
volatility in nearby prices than distant prices. This feature of commodity markets is known as a time-to-delivery effect.

The observed seasonality in demand and storage also creates nonlinear price volatility dynamics. Heating oil prices tend to be more volatile in December through March because the high marginal cost of production together with price inelastic demand means that demand and supply shocks of even a small magnitude can cause a large price swing. Relatively high inventory in early winter allows short-term demand shocks to be partially absorbed though releasing inventory. Such flexibility is inevitably low toward the end of winter and early spring when inventory is low. For unleaded gasoline, the price tends to be more volatile in July and August when demand peaks and in October and March when inventory is the lowest due to the switching of product specifications. For both commodities, the inter-temporal price linkage weakens at the end of demand season when discretionary inventory is low and, for motor gasoline, when the commodity specification changes. Thus, short-term market shocks prior to these low-inventory periods should have small effects on the prices of futures contracts for post season delivery. Put differently, the magnitude of the time-to-delivery effect varies substantially by season.

In the next section, we specify a model of futures prices that captures these dynamic features of energy prices. Specifically, it captures four effects: (1) seasonal and temporal variation in the link between futures prices with nearby and distant delivery dates, (2) a time-to-delivery effect with different magnitudes across seasons, (3) seasonal variation in the level of volatility, and (4) time variation in cross-commodity correlation. In addition, the model permits volatility clustering, which has been documented for many commodity markets.

3. ECONOMETRIC MODEL

In commodity futures markets, multiple contracts trade simultaneously. These contracts differ by time to delivery. Each month, some contracts reach delivery and cease to exist, while others are born and begin trading. From an econometric perspective, a set of futures prices presents a potentially large number of partially overlapping time series. The partially overlapping time-series (POTS) model of Smith (2005) models jointly all contracts trading on a given day. It is a factor model for partially overlapping time series that incorporates time varying conditional heteroskedasticity and time and cross-sectional variation in the factor loadings and innovation variances. It seeks to explain the price changes on all traded contracts with a small number of components, while accounting for time-to-delivery effects, storability, and seasonality.

We extend Smith’s two-factor, single commodity model to a six-factor model of three petroleum commodity prices. We index the price of a futures contract, \( F \), with two subscripts: \( d \) represents the number of trading days until the first day of the delivery month and \( t \) represents the date on which a particular price observation occurs. We use the superscript \( j \in \{ \text{CO, HO, UG} \} \) to denote the
commodity. This \((d, t, j)\) triple is sufficient to identify any price observation in the sample. The model is

\[
\Delta \ln F^j_{d,t} = \theta^j_{1,d,t} \varepsilon^j_{1,t} + \theta^j_{2,d,t} \varepsilon^j_{2,t} + \theta^j_{3,d,t} u^j_{d,t}
\]

where \(\Delta \ln F^j_{d,t} = \ln F^j_{d,t} - \ln F^j_{d+1,t-1}\) denotes the log price change on date \(t\) of the futures contract of commodity \(j\) for delivery at date \(t + d\). The functions \(\theta^j_{1,d,t}\) and \(\theta^j_{2,d,t}\) represent the factor loadings, and \(\theta^j_{3,d,t}\) represents the innovation standard deviation, each of which are deterministic functions of \(d\) and \(t\). The factors \(\varepsilon^j_{1,t}\) and \(\varepsilon^j_{2,t}\) capture the common components across delivery dates, and the idiosyncratic term \(u^j_{d,t}\) captures the component of \(\Delta \ln F^j_{d,t}\) that is uncorrelated with the factors. If the two factors were to explain all of the variation in observed prices, then \(u^j_{d,t}\) would equal zero for all \(t\). In essence, the POTS model uses two unobserved random factors to explain as much of the observed variation in prices as possible. It gains its richness by permitting the factor loadings, which connect the factors to observed prices, to be flexible functions of the season and time-to-delivery. It also gains richness by allowing both the conditional volatility of the factors and their conditional correlations to be time-varying.

We define the first factor for each commodity to be a long-term factor and the second factor to be a short-term factor. We generate this interpretation by setting \(\theta^j_{2,d,t} = 0\) for contracts that are far from delivery, i.e., a short-term shock today cannot affect contracts that mature on a distant date in the future. The condition is equivalent to the one used by Schwartz and Smith (2000) to identify their two-factor model of commodity price dynamics.

Stacking all observed prices into a single vector, the model can be written as

\[
\Delta \ln F = \Theta_{1,t} \varepsilon_{1,t} + \Theta_{2,t} \varepsilon_{2,t} + \Theta_{3,t} u_t
\]

or, in long form as

\[
\begin{bmatrix}
\Delta \ln F^{CO} \\
\Delta \ln F^{HO} \\
\Delta \ln F^{UG}
\end{bmatrix} = 
\begin{bmatrix}
\Theta^{CO}_{1,t} & 0 & 0 \\
0 & \Theta^{HO}_{2,t} & 0 \\
0 & 0 & \Theta^{UG}_{3,t}
\end{bmatrix}
\begin{bmatrix}
\varepsilon^{CO}_{1,t} \\
\varepsilon^{HO}_{2,t} \\
\varepsilon^{UG}_{3,t}
\end{bmatrix}
+ 
\begin{bmatrix}
\Theta^{CO}_{1,t} & 0 & 0 \\
0 & \Theta^{HO}_{2,t} & 0 \\
0 & 0 & \Theta^{UG}_{3,t}
\end{bmatrix}
\begin{bmatrix}
\theta^j_{1,d,t} \\
\theta^j_{2,d,t} \\
\theta^j_{3,d,t}
\end{bmatrix}
\begin{bmatrix}
\varepsilon^j_{1,t} \\
\varepsilon^j_{2,t} \\
\varepsilon^j_{3,t}
\end{bmatrix} + 
\begin{bmatrix}
\varepsilon^{CO}_{1,t} \\
\varepsilon^{HO}_{2,t} \\
\varepsilon^{UG}_{3,t}
\end{bmatrix}
\begin{bmatrix}
u^{CO}_{t} \\
u^{HO}_{t} \\
u^{UG}_{t}
\end{bmatrix}
\]

In (1), \(\Delta \ln F_t\) is an \(n_t \times 1\) vector of daily futures log price changes observed on date \(t\) where \(n_t = n_t^{CO} + n_t^{HO} + n_t^{UG}\) with \(n_t^{j}\) representing the number of futures prices of commodity \(j\) observed on \(t\). The \(n_t^{j} \times 1\) sub-vector, \(\Delta \ln F^j_{d,t}\), is comprised of \(\Delta \ln F^j_{d,t}\). We denote the factors by \(\varepsilon_t = [\varepsilon_{t}^{CO} \varepsilon_{t}^{HO} \varepsilon_{t}^{UG} \varepsilon_{t}^{CO} \varepsilon_{t}^{HO} \varepsilon_{t}^{UG}]^\prime\), which is
a $6 \times 1$ vector of latent factors with $E[\epsilon_i] = 0$ and $E[\epsilon_i \epsilon_j'] = \Omega$ where $\Omega$ is the unconditional correlation matrix with diagonal elements of unity and off-diagonal elements $s_{jk} (j \neq k)$. The idiosyncratic term $u_i$ is an $n_i \times 1$ vector with $u_i \sim N(0, I_{n_i})$ where $I_{n_i}$ is an identity matrix of dimension $n_i$. For identification, we assume $E[\epsilon_t', u_t'] = 0$ for all $i, j, k, d$, and $t$. These assumptions together imply that $E[\Delta \ln F_t | \mathcal{Y}_{t-1}] = 0$ where $\mathcal{Y}_{t-1}$ denotes the information set available at $t-1$. That is, a series of daily futures price changes follows a martingale difference sequence, which implies a zero risk premium.\(^5\)

For the two factors, $\Theta_{1,t}$ and $\Theta_{2,t}$ are $n_t \times 3$ block diagonal loading matrices. For the idiosyncratic term, $\Theta_{3,t}$ is an $n_t \times n_t$ diagonal matrix that determines the idiosyncratic standard deviation. We specify the components of $\Theta_{i,t}$ as deterministic functions of the delivery month of contract ($m = 1, \ldots, 12$) and time to delivery ($d$). This setup allows a particular shock to affect prices differently depending on time to delivery and season. For example, short-term shocks such as hurricanes have a greater effect on nearby prices than those with more distant delivery dates. We consider the following three specifications,

\begin{align*}
\theta_{1,i,t,d}^{(t)} &= \exp(\lambda_{1,i}^{(t)}(d)) \quad \ldots \text{Spec 1} \quad (2a) \\
\theta_{1,i,t,d}^{(t)} &= \exp(\lambda_{1,i}^{(t)}(m,d)) \quad \ldots \text{Spec 2} \quad (2b) \\
\theta_{1,i,t,d}^{(t)} &= \exp(\lambda_{1,i}^{(t)}(d))\exp(\lambda_{2,i}^{(t)}(t))(1 + \exp(\lambda_{2,i}^{(t)}(t)))^{-1} \quad \ldots \text{Spec 3} \quad (2c)
\end{align*}

where

\begin{align*}
\lambda_{1,i}^{(t)}(d) &= a_{i,0}^{(t)} + a_{i,1}^{(t)}d + \sum_{k=1}^{K} \left( a_{i,2k}^{(t)} \sin \left( \frac{2\pi kd}{d_{\text{max}}} \right) + a_{i,2k+1}^{(t)} \cos \left( \frac{2\pi kd}{d_{\text{max}}} \right) \right) \\
\lambda_{2,i}^{(t)}(m,d) &= a_{i,m,0}^{(t)} + a_{i,m,1}^{(t)}d + \sum_{k=1}^{K} \left( a_{i,m,2k}^{(t)} \sin \left( \frac{2\pi kd}{d_{\text{max}}} \right) + a_{i,m,2k+1}^{(t)} \cos \left( \frac{2\pi kd}{d_{\text{max}}} \right) \right) \\
\lambda_{1,1}^{(t)}(d) &= a_{1,0}^{(t)} + a_{1,1}^{(t)}d + \sum_{k=1}^{K} \left( a_{1,2k}^{(t)} \sin \left( \frac{2\pi kd}{d_{\text{max}}} \right) + a_{1,2k+1}^{(t)} \cos \left( \frac{2\pi kd}{d_{\text{max}}} \right) \right) \\
\lambda_{2,1}^{(t)}(t) &= \sum_{k=1}^{K} \left( b_{1,2k}^{(t)} \sin \left( \frac{2\pi ks(t)}{365} \right) + b_{1,2k+1}^{(t)} \cos \left( \frac{2\pi ks(t)}{365} \right) \right)
\end{align*}

In all three specifications, $d_{\text{max}}$ is the maximum days to delivery for which the model is estimated, and, in Spec 3, $s(t)$ is day in year (calendar day) of $t$.

\(^5\) We also estimated a model in which the factors and the idiosyncratic errors were allowed to have non-zero mean. The estimated mean parameters were very small for all three commodities and had no effect on the results presented in the remainder of the paper.
The three specifications allow different levels of flexibility in modelling the volatility dynamics of the three commodity prices. Spec 1 is the most restrictive of three specifications. It specifies the factor loadings and the variance of the idiosyncratic error as functions of time-to-delivery only and assumes each of three functions \((i = 1, 2, 3)\) to be common to all contract delivery months \((m)\). Spec 1 does not allow for seasonality. Spec 2 is the most flexible of the three specifications. Like Spec 1, it specifies the factor loadings and the variance of the idiosyncratic error as functions of time-to-delivery, yet it specifies a different set of three functions for each delivery month. Thus, Spec 2 places no restrictions on how season and time-to-delivery interact to determine volatility.

Spec 3 is an intermediate specification that scales volatility by the same seasonal factor regardless of time to delivery. It specifies the factor loadings and the variance of the idiosyncratic error as products of two components. The first component, \(\lambda_i\), captures the time-to-delivery effect whereas the second component, \(\exp(\lambda_i(d))\), captures seasonal variation. To identify the second factor as the short-term factor, we set \(\alpha_i h_i = 0\) on the second factor is zero at \(d = d_{\text{max}}\). We impose this constraint as

\[
a_{i,0}^2 = -a_{i,1}^2 d_{\text{max}} - \sum_{k=1}^{K} a_{i,2k+1}^2 - \lambda_{\text{min}}, \quad a_{i,0,m}^2 = -a_{i,m,2k+1}^2 - \sum_{k=1}^{K} a_{i,2k+1}^2 - \lambda_{\text{min}}, \quad \text{and}
\]

\[
a_{i,0}^2 = -a_{i,1}^2 d_{\text{max}} - \sum_{k=1}^{K} a_{i,2k+1}^2 - \lambda_{\text{min}}, \quad \text{respectively, for three specifications.}
\]

We set \(\lambda_{\text{min}} = 10\) so that, for all three specifications, \(\theta_{2,d_{\text{max}},t} = \exp(\lambda_{i}(d_{\text{max}})) = \exp(-10) = 0\).

For the conditional variance of \(\epsilon_t\), we specify a multivariate GARCH(1,1) model with dynamic conditional correlation (DCC), as in Engle (2002) and Engle and Sheppard (2001),

\[
E[\epsilon_t, \epsilon_t' | \mathcal{Y}^{t-1}] = H_t = D_t R_t D_t
\]

where

\[
D_t = \text{diag}(\sqrt{h_{1,t}^{CO}}, \sqrt{h_{2,t}^{CO}}, \sqrt{h_{1,t}^{HO}}, \sqrt{h_{2,t}^{HO}}, \sqrt{h_{1,t}^{UG}}, \sqrt{h_{2,t}^{UG}})
\]

\[
h_{i,t}^{j} = \omega_i + \beta_i h_{i,t-1}^{j} + \alpha_i' E[|\epsilon_{i,t-1}^{j}|^2 | \mathcal{Y}^{t-1}]
\]

\[
R_t = (1-\gamma_1-\gamma_2)\Omega + \gamma_1 \eta_{t-1} \eta_{t-1}' + \gamma_2 R_{t-1}
\]

\[
\eta_t = D_t^{-1} \epsilon_t
\]

The matrix \(R_t = (\rho_{j,k,t})\) is symmetric and positive definite with \(\rho_{j,k,t} = 1\) for \(j = k\) and \(\rho_{j,k,t} = \rho_{k,j,t}\) for \(j \neq k\). Because the unconditional variance of \(\epsilon_{i,t}^{j}\) equals unity, \(\omega_i = 1 - \alpha_i - \beta_i\), \(\forall i\) and \(j\). The conditional expectation in the expression for \(h_{i,t}^{j}\) is

\[
E[|\epsilon_{i,t-1}^{j}|^2 | \mathcal{Y}^{t-1}] = (\epsilon_{i,t-1}^{j}|t-1)^2 + P_{i,t-1|t-1}^{j}
\]
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Specifically, to get Schwartz Model 1, set \( r^2/(\sigma_1^2) = 0 \), \( d(0) = 0 \), and \( c(t) = c \); to get Schwartz Model 2, set \( \kappa = 0 \) and \( c(t) = c \); to get Schwartz Model 3, set \( \kappa = 0 \), \( c(t) = c \), and add an interest rate diffusion; to get Casassus and Collin-Dufresne’s model, set \( c(t) = c \), and add an interest rate diffusion.

where

\[
\begin{align*}
\epsilon^i_{t,t-1} & = E[\epsilon^i_{t,t-1} | \mathcal{F}^{t-1}] \quad \text{and} \quad P^i_{t,t-1} = E[\epsilon^i_{t,t-1}^2 - \epsilon^i_{t,t-1} | \mathcal{F}^{t-1}],
\end{align*}
\]

which are obtained through the Kalman filter (Hamilton, 1994).

The POTS model in (1)–(4) is similar to some other factor models of commodity price dynamics. See Lautier (2005) for a comprehensive review of the application of these models to commodities and Gibson and Schwartz (1990), Cortazar and Naranjo (2006), Schwartz (1997), Todorova (2004), and Schwartz and Smith (2000) for applications to crude oil. As exemplified by Schwartz (1997), these authors begin by specifying a continuous-time process for the spot price, before imposing the absence of arbitrage and deriving an expression for the futures price. To connect the POTS empirical model to these Schwartz-type models, suppose the spot price follows a diffusion process under the risk-neutral measure:

\[
\begin{align*}
\frac{dS(t)}{S(t)} &= (c(t) + \delta(t) - \kappa \ln S(t))dt + \sigma_1 dW^Q_1(t) \\
\frac{d\delta(t)}{dt} &= -\alpha \delta(t)dt + \sigma_2 dW^Q_2(t) \\
dW^Q_3(t) &= \rho dW^Q_1(t) + \sqrt{1 - \rho^2} dW^Q_2(t)
\end{align*}
\]

where \( S(t) \) denotes the spot price, \( W^Q_j(t) \) for \( j = 1, 2, 3 \) denote standard Brownian motion processes, \( dW^Q_1(t)dW^Q_2(t) = 0 \), \( \delta(t) \) is a stochastic drift term (often termed convenience yield), and \( c(t) \) denotes a deterministic drift term representing cost of carry. For example, \( c(t) \) could vary over time to account for seasonal differences in storage costs and convenience yield.

This model nests Models 1 and 2 of Schwartz (1997) and differs from Schwartz Model 3 and the model in Casassus and Collin-Dufresne (2005) in that it does not include a stochastic interest rate process in the drift. The absence of arbitrage implies that the futures price equals the expected spot price under the risk-neutral measure. Solving the requisite partial differential equation as in Schwartz (1997) yields

\[
F(t,T) = \exp\left(e^{-\kappa(T-t)} \ln S(t) \right) - \frac{1}{\kappa - \alpha} \left( e^{-\kappa(T-t)} - e^{-\kappa(T-t)} \delta(t) + g(t,T) + \int_t^T c(s)ds \right)
\]

where \( g(t,T) \) is a deterministic function. The time \( t \) increment in the futures price is therefore

6. Specifically, to get Schwartz Model 1, set \( \sigma_2 = 0 \), \( \delta(0) = 0 \), and \( c(t) = c \); to get Schwartz Model 2, set \( \kappa = 0 \) and \( c(t) = c \); to get Schwartz Model 3, set \( \kappa = 0 \), \( c(t) = c \), and add an interest rate diffusion; to get Casassus and Collin-Dufresne’s model, set \( c(t) = c \), and add an interest rate diffusion.
A POTS model in this setting would estimate equation (7) using a panel of partially overlapping time series, with one series for each delivery date $T$. In contrast, the empirical models in the literature focus on direct estimation of equation (6) using a panel that contains one time series for each delivery horizon $T-t$. One advantage of the POTS framework is that the time-varying deterministic component of the drift ($c(t)$) does not appear in (7), even though it is likely to be important in seasonal commodities. Thus, our model is robust to the presence of these components, whereas many Schwartz-type models assume them to be constant. Equation (7) also reveals that mean reversion in spot prices, which is captured by the parameters $\kappa$ and $\alpha$, manifests as an exponential decline in the factor loadings $\alpha_1(t,T)$ and $\alpha_2(t,T)$.

The POTS model in (1)–(4) is more general than (7) because it permits time variation in $\sigma_1$ and $\sigma_2$ in the form of a GARCH process, it models explicitly the idiosyncratic component, and it places no parametric structure on $\alpha_1(t,T)$ and $\alpha_2(t,T)$. In particular, by allowing the data to determine the shape of the factor loading functions, we can incorporate seasonality in the factor loadings. Finally, the POTS model in (1)–(4) identifies the factors differently than the model in (7). In our POTS model, we follow Schwartz and Smith (2000) by labeling the factors as long- and short-run shocks. In contrast, the two factors in (7) represent the spot price and convenience yield.

4. DATA AND ESTIMATION

4.1. Data

We estimate the model in (1)–(4) using daily settlement price data from the NYMEX crude oil (CO), unleaded gasoline (UG), and heating oil (HO) futures markets. All prices are converted into natural log. Contracts traded in these markets are defined in monthly blocks and prices are quoted per barrel of crude oil and per gallon of unleaded gasoline and heating oil. Crude oil contracts trade for delivery as far as seven years whereas heating oil and unleaded gasoline contracts trade as far as 18 and 12 months, respectively. These contracts trade until the

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7. Notable exceptions are Sorensen (2002) and Manoliu and Tompaidis (2002), who estimate models with time variation in $c(t)$. 

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### Table 1: SIC Statistics for Model Selection

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Specification 1</th>
<th>Specification 2</th>
<th>Specification 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crude oil</td>
<td>-734588</td>
<td>-740096</td>
<td>-736030</td>
</tr>
<tr>
<td>Heating oil</td>
<td>-700097</td>
<td>-718041</td>
<td>-701555</td>
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<tr>
<td>Unleaded gasoline</td>
<td>-586744</td>
<td>-601916</td>
<td>-589697</td>
</tr>
</tbody>
</table>

**Note:** Numbers presented are the Schwartz Information Criterion calculated for each of the three specifications of the factor loadings and innovation standard deviation in equation (2) and for each of the three commodities.

third business day prior to the 25th calendar day of the month prior to the delivery month for crude oil and until the last business day of the month prior to the delivery month for the two refined commodities. We use data from December 1, 1984, the date that crude oil and unleaded gasoline markets were opened, to August 31, 2010. Because distant delivery contracts are not always actively traded, we exclude contracts of more than 12 months to delivery from our sample. Excluding these observations leaves a sample of 219,516 prices among 959 contracts.

Estimating the model parameters is computationally intensive due to a large sample size and a large number of parameters defining the model. For example, when the number of trigonometric terms \( K \) is set to 3 the model includes 857 parameters for specification 2. Therefore, following Engle (2002), we estimate the model in two steps. First, we construct three sub-models, each comprised of the daily log-price changes of a single commodity, ignoring the correlation coefficients, \( \rho \)'s. Setting \( K = 3 \) and \( d_{\text{max}} = 252 \), the number of trading days per calendar year, we estimate each of three sub-models by the method of Maximum Likelihood. Second, we estimate the DCC model (3), using the predicted values of the six latent factors and their conditional variances from the first step. This procedure yields less efficient but consistent estimates of model parameters (Engle, 2002).

### 4.2. Estimation Results

**Model Selection**

Table 1 shows the Schwartz Information Criterion calculated for each specification of the factor loading functions in (2). For all three commodities, the

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8. The Heating Oil futures contract was introduced in 1978. The Unleaded Gasoline futures contract ceased trading in December 2006 and was replaced by an RBOB contract. RBOB is a wholesale non-oxygenated blendstock traded in the New York Harbor barge market that is ready for the addition of 10% ethanol. We use the unleaded gasoline contract from December 1984 until December 2006 and the RBOB contract from January 2007 through August 2010.
data support Spec 2 over the other two specifications. Spec 2 has the maximum flexibility of the factor loadings and the variance of the idiosyncratic error. Of the two restrictive specifications, Spec 3 is supported over Spec 1. The results indicate that not only does volatility exhibit significant seasonal variation and time-to-delivery effects for three petroleum commodities, but also that the time-to-delivery effect varies across the twelve delivery months. The three specifications in (2) become more flexible as $K$ increases. Although this extra flexibility allows the model to fit the observed data better, it also makes the coefficient estimates more sensitive to extreme observations. By using only six trigonometric terms, we allow sufficient flexibility to capture seasonality and time-to-delivery effects while avoiding excess sensitivity to outliers. Given these results, we focus on the results obtained for Spec 2 and $K=3$ in our subsequent discussions on the volatility dynamics of the NYMEX petroleum futures.

**Seasonality in Total Variance**

Figure 2 plots the unconditional variance of daily returns as implied by the estimated model. They are computed as $\sum_{i=1}^{3} \hat{\theta}_{i,m,m-d}^2$ for each of the three commodities ($j = \text{CO, HO, UG}$), each of twelve delivery months ($m = 1, \ldots, 12$), and $d$ ranging from 0 to 252 days before delivery. The figure reveals four features about the volatility of daily returns of the three commodities. First, in general, the unconditional variance of daily returns increases rapidly as the contract approaches delivery. This feature, called the time-to-delivery effect, implies that shocks to the underlying spot prices are mean-reverting, that is, they are expected to partially dissipate over time.

Second, volatility in the last two months of trading exhibits substantial seasonality for the two refined commodities. For heating oil, volatility in the last two months of trading is higher for the contracts maturing in late winter to early spring than the other contracts. It is particularly high for the December through March contracts because the demand for heating oil peaks in winter months and demand and supply shocks of even a small magnitude can cause a large price swing. Volatility is also high for the April contract because low inventory after the peak demand season requires that, in years with extended cold weather, higher than usual demand be met by contemporaneous supply. For unleaded gasoline, the unconditional variance in the last two months of trading is substantially higher for the September and October contracts because gasoline inventory is depleted by the end of the summer driving season and the commodity specification switches from the summer to intermediate grade in mid-September and then to the winter grade at the end of October.

Third, although the two refined petroleum commodities exhibit different seasonal patterns in their volatility, high volatility of one commodity appears partially transmitted into the other commodity market. The September and October contracts exhibit high volatility not only for unleaded gasoline but also for
Figure 2: Unconditional Variance of Daily Log Returns as Implied by the Estimated POTS Model

(a) Crude Oil

(b) Heating oil

(c) Unleaded gasoline

Note: The unconditional variance of daily log returns, computed as $\sum_{j=1}^{3} (\hat{\theta}_{j,m,m-d})^2$ for each of the three commodities ($j=CO$, $HO$, $UG$) and for each of twelve delivery months ($m=1,\ldots,12$), is plotted for time-to-delivery ($d$) ranging from 0 to 252.
heating oil, even though demand for heating oil is only moderate in these two months. Similarly, despite low winter demand, volatility increases for the January through March gasoline contracts from mid-November to mid-January. This pattern likely reflects the limited flexibility of the production mix. For example, higher than usual September demand for gasoline can be met by increasing production, which is associated with high co-production of heating oil, which in turn affects the heating oil price.

Fourth, seasonal variation in the volatility of heating oil and unleaded gasoline futures in the last few months of trading is reflected weakly in crude oil markets. Expiring heating oil contracts are more than twice as volatile in the winter as in the summer, and expiring gasoline contracts is twice as large in late summer as in early summer. Aside from the January contract, the range is much tighter in crude oil. This observation implies that high price volatility of the two refined commodities is mostly driven by short-term demand and supply shocks to the refinery sector; any associated shocks to refiners’ demand for crude oil is absorbed through adjusting inventory.

**Decomposition of Petroleum Price Variance**

Table 2 presents, for each of the three commodities, the proportion of the total variance explained by the two factors, \( \varepsilon_{1,t} \) and \( \varepsilon_{2,t} \). We calculate this proportion by \( \left( \frac{\sum_t (\hat{\theta}^i_{1,m,m-d}\hat{\varepsilon}_{1,t} + \hat{\theta}^i_{2,m,m-d}\hat{\varepsilon}_{2,t})^2}{\sum_t (\Delta \ln F_{m,t}^i)^2} \right)^{-1} \) for each delivery month \( m = 1, \ldots, 12 \) and by \( \frac{12}{\sum_m \sum_t (\hat{\theta}^i_{1,m,m-d}\hat{\varepsilon}_{1,t} + \hat{\theta}^i_{2,m,m-d}\hat{\varepsilon}_{2,t})^2}{\sum_m \sum_t (\Delta \ln F_{m,t}^i)^2} \) for the overall share. The table shows that the model explains a large share, on average 98.0%, 97.8%, and 96.3%, of the variation in the daily returns of crude oil, heating oil, and unleaded gasoline futures, respectively. The share of the total variance accounted for by the two common factors is almost identical for all twelve crude oil contracts, whereas it is below average for the high demand season in the two refined commodities. Specifically, there is more idiosyncratic variation.
in the January through April heating oil contracts and in the September through November unleaded gasoline contracts.

Figure 3 illustrates, for each of the three commodities and for each of the twelve contracts, how the proportion of total variance accounted for by the two common factors changes over the one year trading horizon. We measure this proportion by for commodity $j$, $m = 1, \ldots, 12$, and $d = 0, \ldots, 252$. For all three commodities and for all twelve contracts, this share drops substantially in the last few months of trading. That is, much of the high volatility in this period is unrelated to the price movements of more distant contracts that are concurrently traded. The share explained by the factors is particularly low in the last few months of trading for the January through April heating oil contracts and the September through November unleaded gasoline contracts. These are the same contracts with the greatest increase in volatility in the last two months of trading, which reveals that this volatility increase comes from sources uncorrelated with the shocks that affect longer-dated contracts.

For heating oil, this period corresponds to the end of the winter peak-demand season when inventory is inherently low. At this time, demand, supply, and other market shocks affect only the prices of contracts maturing before the end of the peak-demand season and not those maturing after the demand season. Similarly, gasoline stock is naturally low in September, which comes at the end of the summer driving season and just before the specification switches from the summer to winter grade. Low inventory during this period weakens the intertemporal price linkage between contracts near delivery and more distant contracts, resulting high volatility of the September through November contracts that is unrelated to the two factors. For both refined petroleum commodities, this result shows that moderate inventory of the physical commodity toward the end of peak-demand season provides only limited price buffering.

Figure 4 decomposes the unconditional variance of futures prices into three sources; short- and long-term factors, and idiosyncratic shocks, and presents how these three components change over the one-year trading horizon. For illustration, the figure presents this decomposition only for the February and September contracts, which exhibit the highest volatility near delivery for heating oil and unleaded gasoline, respectively. Our identification condition specifies that short-term shocks have zero effect on the price for delivery one year hence. It follows that the short-term factor captures intra-year shocks, those that affect prices for delivery within a year, but are uncorrelated with long-term shocks. The idiosyncratic term captures any price movements uncorrelated with the two factors and, as Figures 3 and 4 show, it appears mostly in the last two months of trading.

Figure 4 shows that, for all three commodities, virtually all futures price volatility around six months before delivery and earlier emanates from the long-term factor. The long-term shock exhibits weak seasonality as evident from the fact that the long-term factor contributes a similar amount of price variance to both the February and September contracts. Similar weak seasonality is also observed for the other ten contracts not presented in Figure 4. Moreover, the variance
Figure 3: Share of Price Variation Explained by the Common Factors

(a) Crude oil

(b) Heating oil

(c) Unleaded gasoline

Note: The share of total variance accounted for by the two common factors, as measured by $\sum_{i=1}^{2}(\theta_{i,m,m,d})^2 / \sum_{i=1}^{3}(\theta_{i,m,m,d})^2$ for each of the three commodities ($j = CO, HO, UG$) and for each of twelve delivery months ($m = 1, \ldots, 12$), is plotted for time-to-delivery ($d$) ranging from 0 to 252.
Figure 4: Variance Decomposed into Three Components

(a) Crude Oil

(b) Heating Oil

(c) Unleaded Gasoline

Note: Three sources of the unconditional variance are measured by \( \hat{\eta}_{j,m,m-d}^2 \) for each of the three commodities \( j = \text{CO, HO, UG} \), for each of twelve delivery months \( m = 1, \ldots, 12 \), and for the time-to-delivery \( d \) ranging from 0 to 252. The figure plots how each of the three sources changes over one year of trading horizon for the February \( m = 2 \) and the September \( m = 9 \) contract.

For all three commodities, much of the volatility increase in the last four months of trading emanates from the short-term factor and the idiosyncratic contribution of the long-term factor is of a similar magnitude across the three commodities. These patterns imply that, on average, the long-term factor reflects non-seasonal supply and demand shocks that are mostly transmitted across the commodities.
shock. For crude oil, these two components exhibit relatively little seasonal variation; for all twelve delivery months, volatility from the short-term shock starts increasing around three months before delivery, whereas the volatility of the idiosyncratic error increases more rapidly in the last two months of trading. In contrast, the two components exhibit substantial seasonal variation for heating oil and unleaded gasoline. For the February heating oil contract, volatility from the short-term shock starts increasing in September, around five months before delivery. The same seasonal pattern is also observed for the other winter contracts (December through March) not shown in Figure 4, for which the short-term shock starts increasing as early as July but more rapidly over September through January. For the two post-winter contracts (April and May), the short-term shock becomes significant in mid-December. For the June through November contracts, volatility from the short-term shock starts increasing around three to four months before delivery. The short-term and the idiosyncratic shocks in the last two months of trading are also much more volatile for the winter and spring contracts than for the other contracts.

The observed seasonality is consistent with the seasonal pattern in demand and storage of heating oil. Weather in fall through early winter determines the amount of inventory carried into the winter peak-demand season, yet the inventory accumulated in this period is not carried over to the post peak-demand season in a normal year. Thus, volatility starts increasing for the winter contracts early fall and is represented by the short-term shocks. In the same period, the contracts maturing after the subsequent winter exhibit no significant movement. The January through March contracts exhibit high volatility at the end of winter peak-demand season due to tight demand-supply balance and low inventory. This high volatility is captured by the idiosyncratic error, which we can think of as capturing very-short-term shocks. Demand, supply, and other market shocks in winter also affect the contracts for post-winter delivery because they determine the amount of inventory carried over to early spring in the case of mild winter. These impacts are rather small and are captured by the short-term factor, which starts increasing as early as mid December for the April and May contract.

The short-term and idiosyncratic shocks also exhibit a strong seasonal pattern for unleaded gasoline. For the August and September contracts, volatility from the short-term shock starts increasing around mid March when the commodity specification switches to the summer grade. For the other ten contracts, the short-term shocks do not affect prices until around four months before delivery. Low inventory at the end of the summer driving season means that demand needs to be met only by contemporaneous supply in September. It also weakens the inter-temporal linkage between the September and subsequent contracts. Consequently, the September contract exhibits high volatility that is captured by idiosyncratic shocks.

In sum, we find a significant time-to-delivery effect and seasonality in variance and correlation across concurrently traded contracts, especially for the two refined commodities. The seasonality mirrors the seasonal demand and stor-
Volatility Dynamics and Seasonality in Energy Prices

Table 3: Estimates of MGARCH Coefficients

<table>
<thead>
<tr>
<th></th>
<th>Crude Oil Coefficient</th>
<th>Heating Oil Coefficient</th>
<th>Unleaded Gasoline Coefficient</th>
<th>DCC Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_i^j$</td>
<td>0.0555</td>
<td>0.0317</td>
<td>0.0605</td>
<td>0.0346</td>
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<tr>
<td>$\alpha_j^j$</td>
<td>0.0848</td>
<td>0.1041</td>
<td>0.0694</td>
<td>0.0295</td>
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<tr>
<td>$\beta_i^j$</td>
<td>0.9444</td>
<td>0.0326</td>
<td>0.9394</td>
<td>0.0356</td>
</tr>
<tr>
<td>$\beta_j^j$</td>
<td>0.9151</td>
<td>0.1045</td>
<td>0.9305</td>
<td>0.0279</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td></td>
<td></td>
<td></td>
<td>0.0249</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td></td>
<td></td>
<td></td>
<td>0.9972</td>
</tr>
</tbody>
</table>

Note: These coefficients are estimated sequentially. First, the estimates of coefficients $\alpha_i$ and $\beta_i^j$ ($i = 1, 2$) are obtained by estimating bivariate GARCH model for each of the three commodities ($j = CO, HO, and UG$) separately. Second, the coefficients in DCC process, $\gamma_1$ and $\gamma_2$, are estimated on the predicted values of six latent factors obtained from the first step.

age cycle of each commodity. The time-to-delivery effect produces greater price volatility in near-to-delivery contracts than far-from-delivery contracts. High price volatility of near-to-delivery contracts emanates from idiosyncratic shocks, which exhibit little to no correlation across concurrently traded contracts and negligible serial correlation. These features together imply that firms with exposure to the calendar spread face significant price risk. A firm that stores crude oil faces much greater volatility in the expected nearby price at which it will buy the commodity than the expected distant price at which it will sell. However, this effect is concentrated in the last six months before delivery and is largest in the final two months before delivery for peak season contracts. Thus, hedging the calendar spread achieves the greatest volatility reduction for short-term exposure to February heating oil and September gasoline.

4.3. Volatility Persistence and Cross-Commodity Correlation (GARCH + DCC)

Tables 3 and 4 summarize the estimates of GARCH parameters. In Table 3, the sum of the coefficient estimates of $\alpha_i$ and $\beta_i^j$ is close to one for each of the two factors and for each of the three commodities. The sum of the two estimated coefficients of DCC process, $\gamma_1$ and $\gamma_2$, is also close to one. These estimates indicate that both the conditional variance and conditional correlation of the underlying factors are highly persistent yet stationary.

In Table 4, the estimated unconditional correlation, obtained as a sample correlation of the standardized factors, is high for the long-term factors; 0.89 for CO-HO, 0.87 for CO-UG, and 0.88 for HO-UG pair, whereas it is substantially smaller for the short-term factors; ranging from 0.29 for HO-UG pair to 0.42 for CO-HO pair. Low cross-commodity correlations of the short-term factors relative to the long-term factors imply that the short-term factors capture demand, supply, and other market shocks that are specific to each market. Examples of such market
shocks include outages of refining facilities and, for the case of heating oil, unusually cold weather in the winter peak-demand season. These shocks will naturally generate a sharp increase in the refined product prices, which revert gradually to the normal level as demand and/or supply respond and inventory adjusts to the normal level. Comparing between the two refined commodities, the correlation with crude oil price is higher for heating oil than for unleaded gasoline in both the short- and long-term factors. This difference reflects the fact that crude oil prices are more volatile in winter (see Figure 2) when demand for space heating energy peaks in the Northern Hemisphere, particularly in the European market.

Figure 5 shows the conditional variances and correlations of the latent factors as predicted by the estimated DCC model. To enhance clarity, the figure presents monthly averages of the daily conditional variances and correlations. In panel (a) of Figure 5, the correlations between the long-term factors exhibit only moderate fluctuations around the corresponding unconditional correlation value, although they increased gradually from an average of 0.8 in the early part of the sample to an average exceeding 0.9 in the latter part of the sample. In contrast, the correlations between the short-term factors vary widely, ranging from −0.12 to 0.82 for CO-HO pair, −0.05 to 0.75 for CO-UG pair, and −0.14 to 0.73 for HO-UG pair (panel (b)). High variation in the conditional correlations of the short-term factors is consistent with our previous discussion that the identified short-term factors sometimes represent commodity-specific shocks, such as a change in seasonal demand, and other times represent a common shock such as a refinery outage.

The correlations between the short-term factors show a gradual trend upwards for most of the sample, rising from about 0.2 in 1989 to a high of about 0.6 in 2002. In the ensuing four years, these correlations dropped back to around 0.3, where they remained through the end of the sample. During the same period, the refining margin exhibits a gradual declining trend, as seen in panel (b) of Figure 6. This is due partly to an incremental cost that refineries incurred in

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Table 4: Estimates of Unconditional Correlation of Latent Factors

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Factor 1</th>
<th>Factor 2</th>
<th>Factor 1</th>
<th>Factor 2</th>
<th>Factor 1</th>
<th>Factor 2</th>
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</thead>
<tbody>
<tr>
<td>CO</td>
<td>0.002</td>
<td>0.891</td>
<td>0.078</td>
<td>0.873</td>
<td>0.115</td>
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<td>0.424</td>
<td></td>
<td>0.239</td>
<td>0.340</td>
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<tr>
<td>HO</td>
<td>0.003</td>
<td></td>
<td>0.877</td>
<td>0.161</td>
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</tr>
<tr>
<td></td>
<td>0.148</td>
<td></td>
<td>0.294</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>UG</td>
<td></td>
<td></td>
<td>0.011</td>
<td></td>
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<td></td>
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<td></td>
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</tr>
</tbody>
</table>

Note: Numbers presented are obtained as the sample correlation of the predicted factor values.
Figure 5: Monthly Averages of Conditional Correlations and Conditional Variances Predicted by the Estimated DCC Model

(a) Factor 1 across three commodities

(b) Factor 2 across three commodities

(c) Conditional variance of factor 1

(d) Conditional variance of factor 2

Note: Panels (a) and (b) plot the monthly averages of conditional correlation across three commodities whereas panels (c) and (d) plot the monthly averages of conditional variance of factors 1 and 2 for each of the three commodities, as estimated by the DCC model.
complying with a series of emission reduction regulations introduced in late 1980s onwards (EIA, 1997, 1999, and 2007). Narrower margins may have resulted in crude oil prices moving more closely with the prices of the refined products as both short- and long-term shocks to the former market are passed more directly to the latter markets.

When oil prices began to increase in 2002 (panel (a), Figure 6), the correlation between the short-term factors began declining. Moreover, between 2002 and the end of the sample, the conditional variance of the short-term factor decreased by almost 50 percent, as seen in panels (c) and (d) of Figure 5. This decrease reveals that the prolonged increase in oil prices from 2003–2008 was driven by long-term shocks. Most of the price variation came from long-term shocks and there were few common short-term shocks. This decline of common short-term shocks is reflected in the declining cross-commodity correlation between the short-term factors.

4.4. Crack Spread Dynamics

Decomposition of Crack Spread Variance—Description

In this section, we explore the volatility dynamics of the crack spread, the margin received from producing either or both of the two refined petroleum
products from crude oil. We consider three crack spreads widely considered in the industry (NYMEX, 1999, 2000); \(-3:2:1, -1:1:0,\) and \(-1:0:1,\) where three numbers represent the short (long if negative) position in the CO, UG, and HO markets, respectively. Based on the estimated POTS model, we decompose the variance of each of these portfolios into three components; long-term, short-term, and idiosyncratic shocks, and examine how the magnitudes of these components change by delivery month and time-to-delivery.

Let \(A\) denote a 1 by 3 vector of positions in the three commodity markets. The variance of the crack spread is

\[
V[A\Delta \ln F_{d,t}] = AV[\Theta_{d,t} \xi_t + \Theta_{3,d,t} \mathbf{u}_t] A' = A \Theta_{d,t} \Omega \Theta_{d,t} A' + A \Theta_{3,d,t} \Psi_{d,t} \Theta_{3,d,t} A'.
\]

In (8), \(A\Delta \ln F_{d,t} = \{\Delta \ln F_{d,t}^{CO}, \Delta \ln F_{d,t}^{UG}, \Delta \ln F_{d,t}^{HO}\}\) is a 3 by 1 vector of daily price change of the three commodities on date \(t\) for delivery at \(t + d\). \(\Theta_{d,t} = \{\Theta_{1,d,t}, \Theta_{2,d,t}\}\) is a 3 by 6 matrix where \(\Theta_{i,d,t} (i = 1, 2, 3)\) is a diagonal matrix with \(\{\theta_{i,d,t}^{CO}, \theta_{i,d,t}^{UG}, \theta_{i,d,t}^{HO}\}\) on the main diagonal. \(\Psi_{d,t} = E[\mathbf{u}_d, \mathbf{u}_d']\) is the covariance matrix of the idiosyncratic errors, \(\mathbf{u}_d = \{u_{d,t}^{CO}, u_{d,t}^{UG}, u_{d,t}^{HO}\}\). The two terms in the right-hand side of (8) represent the components of the crack-spread variance that originate in the six common factors and in the idiosyncratic errors, respectively. Although the model assumes \(E[\mathbf{u}_d, \mathbf{u}_d'] = I_n\), the predicted values of the idiosyncratic errors \(\hat{\mathbf{u}}_d\) are weakly correlated across three commodities. In particular, sample correlations obtained for each of twelve delivery months and for each of 12 monthly blocks of days-to-maturity tend to be higher near delivery (around 0.25 for the first monthly block of days-to-maturity) than far-from delivery (below 0.05 for the fourth and higher monthly blocks of days-to-maturity). Although these correlations are all small, we nonetheless estimate \(\Psi_{d,t}\) in the calculations below using these sample correlations of the predicted values of the idiosyncratic errors.

Because the six latent factors are contemporaneously correlated, most of the off-diagonal elements of \(\Omega\) are non zero (see Table 4). In particular, of the 15 unique covariance terms in the matrix \(\Omega\), three terms representing the covariance between the first and second factor of the same commodity are zero while the other 12 terms representing cross-commodity and cross-factor covariances are non-zero. The presence of 12 non-zero covariance terms makes \(\Theta_{d,t} \Omega \Theta_{d,t}'\) difficult to interpret. Therefore, we orthogonalize the covariance matrix of latent factors through the Cholesky decomposition,

\[
\omega \omega' = \Omega
\]

where the lower triangular matrix, \(\omega\), is the Cholesky factor of \(\Omega\). Substituting (9) into (8) yields,
Although not shown in Figure 8, the variance of crack spreads for other ten delivery months exhibits the dynamics similar to the pattern observed for February and September delivery.

\[
V[\Delta \ln F_{d,t}] = \Gamma_{d,t} \Lambda' + A \Theta_{3,d,t} \Psi \Theta_{3,d,t} \Lambda' 
\]  
(10)

where \( \Gamma_{d,t} = A \Theta_{d,t} \omega \).

The Cholesky factors, \( \omega \), and hence, \( \Gamma_{d,t} \) depend on the order of the six latent factors in the vector \( \epsilon_t \). However, of all the possible orderings, only those that place the three long-term factors first preserve our identification condition, i.e., the three factors corresponding to the short-term shocks have zero loading at \( d = d_{max} \). Furthermore, for those orderings that place the three long-term factors first, the squared sum of the first three components of \( \Gamma_{d,t} \) and that of the last three components are insensitive to the ordering of the three long- and short-term factors. These two sums effectively represent the aggregate long- and short-term shocks to the crack-spread variance. Thus, to understand the components of the crack spread, we order long-term factors before short-term factors and report the aggregate of each.

**Decomposition—Results**

Figure 7 shows, for each of twelve delivery dates, how the variance of three common crack-spread portfolios changes as the contracts approach delivery. Four key observations are of particular interest in the figure. First, for all three portfolios, volatility is low at long trading horizons until it starts increasing rapidly at around two months before delivery. Second, for all twelve delivery dates and for all three spreads, the crack-spread variance at one year before delivery is not zero; it hovers around 20 to 30 percent of the variance of the portfolio without an opposite position in crude oil. Thus, although the crack-spread variance is much lower than the variance of an individual commodity, substantial long-horizon crack-spread price risk exists. Third, the two pairwise crack spreads exhibit seasonal volatility patterns similar to that observed for the relevant refined commodity, with volatility in the last three months of trading substantially greater for the September delivery of the CO-UG spread and the February delivery of the CO-HO spread than the other deliveries. The –3:2:1 spread reflects the seasonal patterns of the two pairwise spreads but more of the CO-UG spread than CO-HO spread. Fourth, volatility is generally higher for the CO-UG spread than for the CO-HO spread.

Figure 8 decomposes the variance of each of the three crack spread portfolios into three components; the short-term, long-term, and idiosyncratic shocks. It illustrates, for each of February and September delivery, how these components change over the one-year trading horizon. The figure shows that the low variance of the three spreads in the first several months of trading originates predominantly in the long-term factor. This pattern reflects the result shown in Figure 4 that essentially all of the price variation at horizons greater than six
Note: The unconditional variance of daily log returns to a crack-spread position is calculated as
\[ \text{V[AΔlnF}_m,m-d] = \text{AΩ}_m,m-d'\text{A' + AΩ}_m,m-d'\text{Ω}_m,m-d'\text{A'} \] where A is a 1 by 3 vector of futures positions in three commodity markets, \( \text{Ω}_m,m-d = [\text{Ω}_{CO,m,m-d}, \text{Ω}_{UG,m,m-d}, \text{Ω}_{HO,m,m-d}] \) with \( \text{Ω}_{i,m,m-d} \) representing a 3 by 3 diagonal matrix with its elements \( \text{h}_i \). \( \text{Ω} \) is the unconditional variance of six latent factors, and \( \text{ψ}_m,m-d = \text{E}[\text{u}_m,m-d'] \) is a 3 by 3 covariance matrix of the idiosyncratic errors with \( \text{u}_m,m-d' = [\text{u}_{CO,m,m-d}, \text{u}_{UG,m,m-d}, \text{u}_{HO,m,m-d}] \). The figure plots the unconditional variance of three crack-spread positions: \(-3:2:1, -1:0:1, \) and \(-1:1:0\), where three numbers represent the short (long if negative) position in the CO, UG, and HO markets, respectively.
Figure 8: Variance of Crack Spread Position Decomposed into Three Components

(a) –3:2:1 Crack Spread

(b) –1:0:1 Crack Spread (CO-HO)

(c) –1:1:0 Crack Spread (CO-UG)

Note: The unconditional variance of daily log returns to a crack-spread position is decomposed into three sources: long-term, short-term, and idiosyncratic shock. We implement this decomposition by:
(i) reorder six latent factors so that three long-term factors are placed first and then three short-term factors, (ii) apply Cholesky decomposition to the covariance matrix of reordered latent factors, i.e., $\chi H_1 \chi^T$, and (iii) calculate the aggregate long- and short-term shocks as the sum of the first three components and the second three components of $H_3 \chi$, respectively. The unconditional variance of idiosyncratic shocks is calculated as $A H_3 \chi, m, m - d W_m, m - d H_3, m, m - d / A$. We calculate such variance decomposition for three crack-spread positions: –3:2:1, –1:0:1, and –1:1:0, where three numbers represent the short (long if negative) position in the CO, UG, and HO markets, respectively. The figure illustrates how three components of unconditional variance change over one year of trading horizon for each of the three crack-spread positions and for each of February and September delivery.
months emanates from the long-term factors. Since the long-term shocks are highly correlated across the three commodities, they tend to be passed through and result in relatively low long-horizon variance of the crack-spread portfolios.

As delivery approaches, all three commodities become increasingly subject to the short-term shocks. Although these short-term shocks exhibit only moderate correlation across the three commodities, they are less volatile for crude oil than for the two refined commodities. Consequently, until around two months before delivery, the crack-spread variance decreases gradually relative to the variance of a refined-product portfolio without an opposite position in crude oil. The crack-spread variance is lower for the CO-HO than for the CO-UG spread because the correlation in the short-term factors is higher for CO-HO pair than for CO-UG pair.

In the last two months before delivery, all three commodities become subject to the idiosyncratic shocks, which are only weakly correlated across the three commodities. The overall price correlation between crude oil and the two refined commodities decreases rapidly in this period and consequently the volatility of the crack-spread portfolios is almost as large as for the individual refined commodities. The potential benefits of hedging crack-spread variance with futures contracts are thus magnified during this period. The magnitudes of the idiosyncratic shocks differ substantially across the twelve delivery dates, resulting in large seasonal variation in the benefits of crack-spread hedging in the last one month of trading period.

The –3:2:1 spread exhibits much greater variance than two pairwise spreads because it includes three times the number of contracts as the pairwise portfolios. However, once scaled to one ninth to make it comparable to the pairwise portfolios, the variance of the –3:2:1 crack spread is substantially smaller than the pairwise portfolios, particularly near the delivery date. This low variance arises because this portfolio is more widely diversified and all three commodities are positively correlated in their common factors. The –3:2:1 spread also reflects the seasonal pattern of unleaded gasoline more than that of heating oil, simply because the portfolio contains twice as more units of the former than the latter.

The volatility of the crack spread indicates the potential daily effectiveness of hedging spread price risk. To illustrate this point, consider a refiner who plans to buy crude oil and sell heating oil in December of a particular year. In January of that year, the December futures prices represent the expected spot prices eleven months later. The change in futures prices from day to day equals the change in the expected December price. If the percentage change in the crude oil futures price exactly offsets the percentage change in the heating oil price, then the crack-spread volatility would be zero and there would be no gain to hedging today rather than waiting until tomorrow. On the other hand, positive crack-spread volatility indicates that a firm holding a physical crack-spread position is exposed to price risk. The magnitude of this volatility measures the potential change in the value of the firm’s position from one day to the next.

This point can be further explored through reference to the time series plot of the relative spot prices of the refined commodities to crude oil, as shown
in Figure 6. As often reported (EIA, 1997), these relative prices exhibit seasonal variations, reflecting strong seasonality in demand for refined petroleum commodities. Figure 6 indicates that the relative prices deviate from such seasonal means by substantial magnitudes, indicating considerable price risk of crack-spread positions in short run. Our model reveals that, for all three crack-spread positions and for all twelve delivery dates, the price risk is substantial even at the one year horizon, where it constitutes around 20 percent of the corresponding variance of the underlying commodities. These prolonged fluctuations in the crack spread also comport with the fact that the long-term factors are highly, but not perfectly, correlated across commodities. Thus, substantial benefits remain to hedging crack-spread positions at the one-year horizon.

5. CONCLUSION

We examine the volatility dynamics of the futures prices of three petroleum commodities traded at NYMEX. Using the partially overlapping time-series framework of Smith (2005), we model jointly all contracts with delivery dates up to a year into the future and thereby use the information available from the markets about the persistence of price shocks. We extend Smith’s original specification into a three-commodity, six-factor setting, and decompose the daily futures returns for each commodity into two common factors, short- and long-term, and a contract specific component. The model specifies the factor loadings and the variance of the contract specific shocks using non-parametric functions and allows temporal variation in the conditional variances and correlations of the six factors.

The estimated model reveals highly nonlinear volatility dynamics of the three commodity prices that are consistent with the observed seasonality in demand and storage. For all three commodities, volatility exhibits both time-to-delivery effects and substantial seasonal variations, yet these volatility patterns differ substantially across the three commodities and by delivery month of the contract. For heating oil, the volatility in the last few months of trading is higher for the contracts maturing in late fall through the end of winter, during which the demand peaks for space heating. It is particularly high for the January through March contracts due to low inventory at the end of the peak-demand season. For unleaded gasoline, the volatility is high for contracts maturing in late summer through fall when gasoline inventory is low after the summer peak-demand season and before the commodity specification shifts from the summer to winter grade. For crude oil, relatively weak seasonal variation is depicted in the long-term factor, with volatility slightly higher for all twelve contracts during early fall through winter when demand peaks for spacing heating in the Northern Hemisphere. The same seasonal pattern is observed for the two refined commodities, implying that the long-term shocks to the crude oil market correlate strongly with those of the two refined commodities.

For all three commodities, a large share of price variation in the last several months of trading emanates from the short-term factor and the idiosyn-
Volatile errors. Idiosyncratic shocks are particularly large for heating oil and unleaded gasoline in the last two months of trading on contracts that mature in their respective peak-demand seasons. These contract-specific shocks are very weakly correlated, resulting in low price correlation between contracts maturing before and after the end of peak-demand season. This finding is consistent with the theory of storage, which suggests that low inventory at the end of peak-demand season weakens the inter-temporal price linkage. The correlation is also low earlier in the peak-demand season, implying that high inventory provides only limited buffering of very short-term price shocks.

The short-term and idiosyncratic shocks also exhibit much lower correlation across the three commodities than do the long-term shocks. Consequently, crack-spread portfolio returns are very volatile at short horizons and their variance near delivery is as large as the variance of each refined commodity price without an opposite position in crude oil. This short-run variance is about three times as large during the peak-demand seasons as in the low-demand seasons, implying relatively large benefits to short-horizon hedging during peak-demand periods. High cross-commodity correlation of the long-term factors implies low volatility of the long-horizon crack spreads. However, long-horizon (greater than six months) price risk remains significant, above 20 percent of its level for the individual commodities. These implications for the long-horizon price risk of petroleum commodities, either individually or their cross-commodity spreads, have not revealed before because previous studies have examined only a subset of price data available from the markets and/or because their models have presumed a stochastic process for petroleum commodity prices that is too restrictive to replicate the features implied by the theory of storage.

Our results also have implications for the margin requirements imposed on hedging firms. Just as the seasons and planning horizons with the highest volatility provide the greatest potential benefits of hedging, they also provide the highest probability of default on a hedge position. Default can occur if prices move by more than the margin amount held by the counterparty or clearing house. On the other hand, setting margins too high reduces market liquidity by raising the cost of trading for hedgers as well as speculative traders. Thus, counterparties and clearing houses could enhance efficiency by requiring larger margin at short horizons and during peak demand seasons. These implications will be investigated in further research, especially given that the Dodd-Frank Wall Street Reform and Consumer Protection Act (2010) promises significant changes in the way that energy hedging contracts are constructed.

REFERENCES


