Long-run Cost Functions for Electricity Transmission

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Electricity transmission has become the pivotal industry segment for electricity restructuring. Yet, little is known about the shape of transmission cost functions. Reasons for this can be a lack of consensus about the definition of transmission output and the complexity of the relationship between optimal grid expansion and output expansion. Knowledge of transmission cost functions could help firms (Transcos) and regulators plan transmission expansion and could help design regulatory incentive mechanisms. We explore transmission cost functions when the transmission output is defined as point-to-point transactions or financial transmission right (FTR) obligations and particularly explore expansion under loop-flows. We test the behavior of FTR-based cost functions for distinct network topologies and find evidence that cost functions defined as FTR outputs are piecewise differentiable and that they contain sections with negative marginal costs. Simulations, however, illustrate that such unusual properties do not stand in the way of applying price-cap incentive mechanisms to real-world transmission expansion.

Key words: Electricity transmission, Cost function, Incentive regulation, Merchant investment, Congestion management

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1. INTRODUCTION

Under the restructuring of electricity sectors around the world electricity transmission has been playing a pivotal role. Electricity transmission enables electricity trade within and across countries. It can enhance competition. It can increase the reliability of the electricity system and substitute for lack of generation in certain areas. Congestion and failure of electricity transmission can lead to brownouts and blackouts over large regions. Before electricity restructuring transmission was usually vertically integrated with often large generation companies and sometimes also with distribution companies. More recently, independent transmission entities (Transcos) have been emerging.

All these developments have raised interest in electricity transmission services and led to extensive research studying the economic properties of transmission systems. We know that electricity transmission differs in many ways from other transportation systems, such as pipelines, railroads or the road system, and from other network industries, such as telecommunications. The physical laws of electricity make transmission complex and unusual. It is therefore not surprising that—to the best of our knowledge—no one so far has characterized a cost function for transmission grids. Two specific reasons appear to be responsible for this lack. One is that there is no full agreement on an obvious output, to which costs could be related. Thus, there may exist cost characterizations for the transmission grid as a set of line capacities, but, in our view, such capacities are definitely not the transmission outputs. The second reason for a lack of cost function characterizations is that the effects of Kirchhoff’s laws lead to bewildering irregularities in the relationship between outputs and capacities. An example of such irregularities is the famous Wu et al. (1996) paper on (the pitfalls of) folk theorems on transmission access. As a result transmission cost functions are likely to have strange properties that would make them interesting for an audience outside electricity. One such property that we expected from the outset is that instances of negative marginal costs might occur.

Thus, the current paper provides a characterization of the general shapes of transmission cost functions, (based on a more rudimentary earlier attempt in Hogan, Rosellón and Vogelsang, 2010; in the following: HRV).


2. A referee pointed out to us that there is nevertheless a long history of formulations of optimal transmission planning in the context of, for example, given generation and demand requirements. Even though such formulations pre-date the invention of FTRs, these models effectively represent the transmission planning problem in terms of the injections and withdrawals paradigm of FTRs. Such formulations propose to find the minimum cost (or achieve some other objective) transmission expansion plan, in order to provide transmission service from given generation to demand, given assumed line costs. For example, see Villasana (1984).

3. Using data from several standard references, Baldick and Kahn (1993) had already pioneered the numerical estimation of transmission cost functions. They derived them plotting data on trans-
Besides satisfying an intellectual curiosity the knowledge about properties of such cost functions can be put to use, among others, as a planning tool for Transcos and regulators. If one knows their cost functions one can plan cost-minimizing transmission systems over a wide range of potential outputs. This would also take care of reliability issues requiring the availability of alternative paths in case of network failures or of unplanned electricity injections at some nodes because of generation failure at other nodes. The knowledge of a transmission cost function could also help assess investments in renewable energies, such as wind power, that may require substantial transmission investments. In addition, the knowledge of transmission cost functions can aid the implementation of incentive mechanisms for transmission investment. We will demonstrate this role specifically for the HRV mechanism, but it should in principle hold as well for Bayesian mechanisms by using techniques similar to Gasmii et al. (2002).

The regulatory analyses of incentives for electricity transmission expansion postulate transmission cost and demand functions with fairly general properties, and then adapt regulatory adjustment processes to the electricity transmission expansion problem. Under well-behaved cost and demand functions (and assuming a natural monopoly), appropriate weights (such as Laspeyres weights) grant convergence to equilibrium conditions (Vogelsang, 2001, Tanaka, 2007, Rosellón, 2007, and Léautier, 2000). A criticism of this approach is that the properties of transmission cost and demand functions are little known but are suspected to differ from conventional functional forms (Hogan, 2000, 2002a, Vogelsang, 2006, and HRV, 2010). Hence the assumed cost and demand properties may not hold in a real network with loop-flows since decreasing marginal cost segments and discontinuities in the costs can arise during an expansion project. Furthermore, a conventional linear definition of the transmission output—which is in fact difficult since the physical flow through loop-flowed meshed networks is complex and highly interdependent among transactions (Bushnell and Stoft, 1997, and Hogan, 2002a, 2002b).

In this paper, we study long-run electricity transmission cost functions based upon a definition of transmission output in terms of point-to-point trans-

mission costs against capacity, and considering economies of scale, lumpiness, and reliability. They even further addressed the issue of the trade-off between generation and transmission expansion as well as the implied regulatory problem of achieving welfare-optimal transmission (and generation) planning. They also found that the assumption of a wholesale competitive structure in the generation sector implies inefficient transmission planning due to information asymmetries. Typically, competition in generation would imply less transmission inefficiencies when generation capacity is dispatched in the baseload mode.

4. In this paper, a cost function with increasing returns to scale and declining marginal costs is considered well behaved, while one with negative marginal costs and kinks (in an otherwise smooth function) is not.

5. Dismukes et al. (1998) empirically show the validity of the natural-monopoly assumption for electricity transmission.
actions or financial transmission right (FTR) obligations. We build on the HRV (2010) model which combines merchant and regulatory approaches in an environment of price-taking generators and loads.\(^6\) The HRV model also shows that FTR-based cost functions exhibit very normal economic properties in a variety of circumstances. This particularly holds if the topology of all nodes and links is given and only the capacity of lines can be changed, implying that abnormally behaving cost functions require changes to network topology. We study in more detail these conclusions, and test the behavior of FTR-based cost functions for distinct network topologies. We focus on two basic cases. In the first we adjust line capacities, but nodes, lines, impedances and thus the power transmission distribution factors (PTDFs) do not change.\(^7\) This framework allows us to single out the effect of loop-flows on transmission costs. In the second case we allow for changes in line impedances (and thus the PTDFs) correlated to the changes of line capacities. These cases provide insights about the relationship between PTDFs, transmission capacity, and transmission costs.

Our model builds on FTRs as meaningful transmission outputs that allow for the characterization of a transmission cost function as the minimum cost locus over a range of outputs. This model seeks to overcome current limitations in regulating electricity transmission networks by relying on general microeconomic foundations. This represents a potentially significant improvement in regulation over status-quo regulatory practices. Actual regulation of access to transmission networks, such as the cost of service regulation schemes predominantly applied in the U.S., is mostly cost based but relies on no cost modelling. In particular, unlike telecommunications, electricity regulators make little reference to cost minimization for a well-defined output. Likewise, transmission regulators engage in transmission expansion planning without a theory-based planning tool. Our model is therefore also a first necessary step towards improved incentive regulation schemes for transmission network expansion which import the main concepts of regulatory-economics theory into electricity-grid regulation.

Furthermore, the cost analysis carried out in this paper contributes to providing improved sound estimates of actual transmission network costs. To our knowledge, except for the planning literature mentioned in Footnotes 2 and 3, this issue has hardly been addressed in the literature. There exist some studies of issues related to network cost recovery from an analytical perspective (e.g., Pérez-Arriaga et al., 1995, Green, 1997, Lima et al., 2009, and Olmos and Pérez-Arriaga, 2009), while other studies analyze cost allocation to network users according to actual impacts on power flows (e.g., Kattuman et al., 2004, and Olmos and Pérez-Arriaga, 2007). Likewise, Breseti et al. (2009) have analyzed the quantification of the benefits from network extensions, including reliability aspects, and impacts on the competition within the market. Nevertheless, most studies of network costs

\(^6\) The model is an extension of Vogelsang (2001) for meshed projects. While designed for Transcos, it can be applied under an ISO setting.

\(^7\) See section 3.1.
do not assess the actual functional form of electricity transmission costs. In contrast, this paper proposes a first methodological step towards empirical cost estimations for transmission networks based on simulations (in the tradition of Gasmi et al, 2002).

The plan of the paper is as follows. In Section 2 we discuss the characterization of transmission outputs in terms of FTRs. In Section 3 we address the mathematical cost-function model as well as its adaptation to a computer programming model. In this section, we also describe the dataset used to make simulations as well as the different functional forms to be tested. The results of simulations are presented and discussed for fixed and variable line reactances in Section 4. This section also identifies the challenges associated with consideration of the effects on cost functions of changes in network topology. In Section 5 we illustrate through simulation results how the properties we found for transmission cost functions are conducive to the functioning of the HRV regulatory mechanism. Section 6 concludes.

2. CHARACTERIZATION OF ELECTRICITY TRANSMISSION OUTPUTS

In a vertically separated setting with transmission provided by a stand-alone Transco, the grid is used by generators that want to deliver electricity to load-serving entities (loads, or LSEs), and by entities that want to purchase from generators with or without the help of intermediaries. Transmission makes these transactions possible. Thus, the Transco’s chief service is to provide delivery between generation nodes and consumption nodes. Bushnell and Stoft (1997), and Hogan (2002a, 2002b) argue that the definition of the output for transmission is difficult since the physical flow through a meshed transmission network is complex and highly interdependent among transactions.

Under a network with loop flows, outputs could be defined as bilateral trades between pairs of nodes that aggregate to net injections at all nodes. This idea derives from the FTR literature which does not consider transmission activity as an output (or throughput) process, but instead concentrates on “point-to-point” (PTP) financial transactions based on rights, obligations and options (Hogan, 2002b). Physical transmission rights are also discussed in the FTR literature. However, as mentioned above tracing the physical flow “without consideration of the effect of all flows on binding constraints cannot reveal which collections of flows are collectively feasible with respect to those constraints” (anonymous referee). The superiority of FTRs over physical rights has been analytically demonstrated as well (Joskow and Tirole, 2000).10
The difference between an FTR and a physical right can be analyzed via a three-node-network setting (see Figure 1). Assuming equal line impedances, power injected and withdrawn at two nodes within the system (e.g., 9 MW) will cause 2/3 of the energy (6 MW) to flow on the direct connection (n2-n3) and 1/3 (6 MW) will flow over the longer connection (n2-n1, n1-n3). Given a transmission amount of 9 MW the corresponding FTR would be a point-to-point right of 9 MW from node 2 to node 3, whereas the corresponding physical representation would be a 6 MW right for the direct link (n2-n3) and two 3-MW rights for the flow over the other two lines (n2-n1, n1-n3) (Figure 1, case 1). Assuming a second injection-withdrawal pair of 3 MW between node 1 and node 3, the corresponding power flow values will lead to counter flows on the line between node 1 and node 2 (Figure 1, case 2).

The changed market conditions of case 2 have an impact on the underlying physical rights: In case netting is not included in the market design, the physical counter-flow effect will not be taken into consideration, although only 2 MW will flow from node 2 to node 1. For each injection/withdrawal pair it will be required to hold a physical right of 3 and 1 MW respectively. In case netting is permitted, the required physical position of the 9 MW injection would change reducing the required capacity right for the counter flow link to 2 MW. FTRs do not account for a specific power flow pattern, and thus the holder of the 9 MW FTR will not have to alter any position due to changes in the market dispatch. Any impact on the power flow and congestion situation will be fully reflected by changes in the nodal prices.

In this paper, we capture the delivery function of electricity among nodes via FTRs that are defined between nodes. An FTR $q_{ij}$ represents the right to inject electricity in the amount of $q$ at node $i$ and to take delivery of the same amount

“unbalanced”. A perfect hedge is achieved through a balanced PTP-FTR (providing the same injection and withdrawal at different locations), while an unbalanced PTP-FTR obligation can be seen as a forward sale of energy at any location of the system. See also Hogan (2002b).
at node \( j \) (this definition for FTRs works for obligations, as opposed to other hedging instruments such as options). The FTR does not specify the path taken between \( i \) and \( j \). It is a flow concept and therefore applies to a discrete point in time and to PTP transactions.

Therefore, to analyze the cost behavior of extending meshed networks we define the transmission output as PTP transactions. Whereas in directed networks like natural gas or oil an additional unit of output normally can be associated with a well-defined cost parameter or function, additional output in electricity networks depends on the grid conditions, and cannot be considered separately from the output setting.

3. MODEL, TOPOLOGIES AND DATA

3.1 The FTR Cost-function Model

One purpose of our study is to establish that the problem of non well-behaved, non-continuous transmission cost functions is related to demand changes that lead to a change in network topology (as suggested by HRV, 2010). We restrict our current analysis to cases where the network topology is not changed, first studying cases with no changes in impedances, and then addressing the effects of loop-flows in switched networks.

We define network topology as a set of nodes and their locations and a set of lines between nodes. The network topology is described by the network incidence matrix \( I_{nl,i} \), defining which nodes \( i \) are connected by lines \( l \) (e.g., see Léautier, 2000). Given the topology there is a set of power transfer distribution factors (PTDF) that represent the share of a power transfer that flows on a specific line. Generation nodes and consumption nodes are naturally given by the set of transmission outputs (FTRs), while free nodes are deliberately chosen for optimization of the network topology. A three-node network could be associated either with three lines connecting all three nodes, with three possible combinations of two lines, or with three possibilities of one line. Obviously, in the cases of a single line one node would be an orphan and could not be used for injecting or consuming electricity. For a given network topology we assume that the line capacity is variable so that it can be changed between 0 and \( \infty \), but at a cost. There may be a fixed cost at zero capacity.

To derive an FTR-based cost function for transmission we examine the properties of power-flows in meshed networks. We use the DC load-flow model (DCLF) as proposed by Schewppee et al. (1988), which focuses on real power-

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11. An anonymous referee pointed out that in the UK system the transmission charging mechanism assumes expansion only along existing lines.

12. The incidence matrix has a value of +1 if the node \( i \) is the starting node of a line, and a −1 if it is the end node of a line. Each line connects exactly two nodes thus each row of the incidence matrix sums to zero.
flows and neglects reactive power-flows within a network. Although a simplification, the approach still yields reasonable results for locational price signals and grid utilization (see e.g., Overbye et al., 2004). The complete approximation of the DCLF from the physical fundamentals of transmission lines is presented e.g., in Stigler and Todem (2005). 13

Based on the DCLF power flows on a line \( l \) can be derived by using the PTDF matrix:

\[
pf_i = \sum_{l} ptdf_{i,l}n_i
\]

The PTDF matrix allows us to obtain the power flow on a line \( l \) by a linear function depending on all net injections \( n_i \). However, the PTDF matrix itself depends on the underlying network topology \( In_{l,i} \) and the total set of all line characteristics represented by the series susceptances \( B_l \). 14

The power-flow on one line also has an impact on the energy balance of its connected nodes. For each node \( i \) in a system the net injection \( n_i \) must equal the sum of power-flows on connected lines as indicated by the network topology \( In_{l,i} \):

\[
n_i = \sum_{l} In_{l,i}pf_l
\]

If more energy is to be delivered to or from node \( i \) all power-flows and nodes on lines connected to that node are affected, continuing throughout the network. Therefore, the resulting power-flow pattern depends on all system conditions.

To assess the costs of transmission, we define the transactions \( q_{ij} \) between two nodes \( i \) and \( j \) as the relevant output. 15 These FTR PTP transactions are determined as a specific load value, e.g., in MW that must be transmitted between the two nodes. There is no pre-specified line utilization associated with an FTR. Market participants can bid for specific FTRs and the system operator allocates them accordingly, maximizing the revenue from the FTRs given the network’s available transmission capacity. FTRs are assumed to be obligations, thus the associated energy transfer can be taken for granted.

We define the transmission costs function \( c(\cdot) \) of network extension as the least costs combination of line capacities \( k \) necessary to satisfy \( Q_{ij} \) (the matrix

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13. The principle of a DCLF is that flows \( p_{fi} \) on a line \( l \) depend on the voltage angle difference \( \Theta_l \) between the two nodes \( i \) and \( j \) connected by that line and the line series susceptance \( b_l \): \( p_{fi} = b_l \cdot \Theta_l \).

14. Variables/Parameters that represent a matrix are written with capital letters (e.g.: \( B_l \) is the matrix of all specific line series susceptances \( b_l \)).

15. An FTR \( q_{ij} \) is defined by two net injections \( n_i \) and \( n_j \) that have opposite signs: \( q_{ij} = (\ldots - n_i, \ldots + n_j, \ldots) \). Which in turn means that the net injection at a node is a function of all FTRs within the system \( n_i(Q_{ij}) \).
Long-run Cost Functions for Electricity Transmission / 139

consisting of a specific set of FTR combinations \( q_{ij} \) given the network topology \( In_{il} \):\(^{16}\)

\[
c(Q_{ij}) = \min \sum_{k_i} f(k_i) \]

transmission cost function \( (3) \)

Thus, our approach is one of long-run cost functions, where capacity is optimally adjusted to each output.\(^{17}\) In equation (3) each line capacity \( k_i \) connecting two nodes \( i \) and \( j \) is associated with a specific cost value via an extension function \( f(.) \).

Minimization is subject to technical restrictions representing the network’s power-flow characteristics:

\[
|pf_l| = \left| \sum_{i} ptdf_{i,i} ni_i \right| \leq k_i \quad \forall l \]

line capacity constraint \( (4) \)

\[
\sum_{j} q_{ij} - \sum_{j} q_{ji} = ni_i = \sum_{l} In_{il} pf_l \quad \forall i \]

energy balance constraint \( (5) \)

First, power-flows \( pf_l \) on the line \( l \) must remain within the capacity limits \( k_i \) defined by the system operator when designing the grid (equation 4). Second, at each node \( i \) the sum of outgoing FTRs \( q_{ij} \) and ingoing FTRs \( q_{ji} \) which equals the net injection \( (ni_i) \) must equal the sum of power-flows on connected lines \( pf_l \) (equation 5).

Let \( i = 1, \ldots, I \), \( j = 1, \ldots, J \), and \( l = 1, \ldots, L \). If we assume that no line capacity is wasted,\(^{18}\) the installed capacity will equal the power flow of that line (equation 4 holds with equality) which gives us:

\[
\min \sum_{l} f(|pf_l|) = \min \sum_{l} f(|\sum_{i} ptdf_{i,i} ni_i|) \\
= \min f_i(\sum_{i} ptdf_{i,i} ni_i) + f_2(\ldots) + \ldots + f_L(\ldots) \\
= \min f_i(ptdf_{i,i} ni_i + ptdf_{i,i} ni_2 + \ldots + ptdf_{i,i} ni_I) + f_2(\ldots) + \ldots + f_L(\ldots) \quad (6)
\]

where \( ptdf_{i,i} \) is a specific realization of the PTDF matrix. Given our assumption of full line utilization the variables in parenthesis are the \( k_i \)'s, for example:

\[
k_i = ptdf_{i,i} ni_i + ptdf_{i,i} ni_2 + \ldots + ptdf_{i,i} ni_I
\]

\(^{16}\) We are only considering capacity-related costs (no proper operating costs).

\(^{17}\) In practice, transmission capacities can only be changed over long time periods so that they will not be perfectly adapted to demands. Also, costs are subject to uncertainty about input prices, input quantities and technology changes (such as in “smart grids”).

\(^{18}\) This assumption does not hold if we have lumpy investments. Furthermore, the results in case of economies of scale do indicate that this assumption may also be invalid for specific continuous cost function cases.
The power flow is dependent on the PTDF and the net injections which are defined by the FTR matrix $Q_{ij}$. The PTDF matrix is a function of the network topology $In_{ij}$ and the overall line series susceptances $B_i$ which themselves are defined by the line capacities $K_i$.

Costs depend on FTRs directly and indirectly, where the indirect effects involve a modification of susceptances, of the PTDF matrix and of the power flows, all due to the required capacity changes. Since the $f_i(.)$ are the costs of individual lines, they are monotonically increasing functions. The marginal costs of FTRs are linear combinations of the marginal costs of all lines. The weights of these linear combinations are the ptdfs. As a result, marginal costs of FTRs should be well-behaved as long as marginal costs of lines do not differ too widely from each other and as long as marginal costs of all individual lines are well-behaved. However, the opposite might also be said: if marginal costs of each line differ too widely from each other then the marginal costs of FTRs will most likely be ill behaved.\textsuperscript{19}

We will next gather evidence of this theoretical implication with simulations over simple networks for, first, the case of no ptdf change over time and, second, when ptdf changes are allowed. To examine the characteristics of the FTR-based cost function in (3) and restrictions (4) and (5), we incorporate the model in a General Algebraic Modeling System (GAMS) as a non-linear minimization tool, with the overall grid extension costs as an objective function and looped over a specific set of FTRs.

3.2 Network Topologies and Dataset

We use a numeric data set representing idealized market characteristics to test our FTR-based cost function model. We consider two grid topologies to carry out our simulations (Figure 2):

1. An initial grid topology that comprises a three-node network with two generation nodes and one demand node representing the basic loop-flowed network structure.

2. An extended six-node network with two generation nodes and one demand node.

FTRs are defined from each of the two generation nodes to the demand node, and vary between 1 MW and 10 MW respectively, to estimate the resulting global-cost function.

For network extension behavior, we next analyze two cases:

1. Only the capacity of a line can be changed whereas the line’s impedance remains unchanged. Thus, an extension only impacts the trans-

\textsuperscript{19} As the ptdf values can be both positive and negative, the impact on the overall cost function can also be positive and negative. This is also true if it is assumed that there is no linkage between capacity and ptdfs (see below our first model case in section 4.1).
mission capacity of the system, but does not alter the power-flow pattern and PTDF structure. This approach is theoretical, since in reality pure capacity increases are only possible for small-scale extensions. It assesses the impact of loop flows on transmission costs without the interfering influence of power-flow changes.

2. Line extensions are combined with a change of the line’s impedance and the added capacity changes the network’s power-flow pattern as well as the PTDFs. This approach resembles the real world problem that a new or upgraded line affects the entire network: A line specifically built to enable certain FTRs will lead to externalities as it affects the costs of all other FTRs.

We test four forms of line extension costs functions \( f_{ij}(k_{ij}) \): constant marginal cost, decreasing marginal cost (economies of scale), increasing marginal costs (diseconomies of scale), and lumpy behavior.

20. Examples include heat monitoring to allow a higher transmission in case of low temperatures or changes in the security guidelines allowing a higher transmission level on the same line or re-conductoring and line tensioning (Baldick and O’Neill, 2009).

21. Although diseconomies of scale are rather unlikely in electricity networks they are included for the sake of completeness. Diseconomies of scale could arise if environmental costs were included in the cost functions of transmission lines.
Linear function (constant marginal cost): \( f_{ij} = b_{ij} k_{ij} \)

Logarithmic function (economies of scale): \( f_{ij} = \ln(a_{ij} + b_{ij} k_{ij}) \)

Quadratic function (diseconomies of scale): \( f_{ij} = b_{ij} k_{ij}^2 \)

Lumpy function: \( f_{ij} = b_{ij} k_{ij} \) with \( k_{ij} \in Z^+ \)

In order to bring out the qualitative features of the cost functions, we assume in each case that the parameters \( a \) and/or \( b \) of the line cost functions are the same for all lines in the basic setting.\(^{22}\) The first three extension functions represent a continuous approach and the fourth directly accounts for the integer nature of line extensions, which is a rough approximation of the lumpy investment pattern of electricity networks. Real world network extensions represent a combination of lumpy investment behavior with economies of scale.

For all scenarios the network topology is fixed; new connections cannot be built and existing connections cannot be abolished. We assume that each line has the same starting characteristics for capacity and reactance whereas the resistance is assumed to be zero.\(^{23}\)

When only the line capacities are extended, the presented extension cost functions are sufficient to derive a numerical solution. In the case of a connection between extensions and line reactance, the law of parallel circuits is applied to derive a functional connection between capacity extensions and line characteristics \( B_{ij}(k_{ij}) \). Thus, doubling the capacity results in a bisection of a line reactance value. Consequently an initial level of capacities and reactances is needed for this approach making the obtained extension costs the incremental costs from the given start conditions. We test the cases using a series of numerical analyses, varying the underlying parameter. An overview of the basic data set is provided in Table 1.

4. SCENARIOS AND RESULTS

We first present the results for the cases with fixed-line reactances and thus the impact of loop flows on extension costs. To single out the effects we start with the extension of one FTR, while the other is kept fixed, and then allow both FTRs to be extended. We then analyze the case with a linkage of capacity and reactances—case of variable line reactances—to estimate the combined impact on extension costs. We end with a discussion of the results.

\(^{22}\) This is a strong but necessary simplification in order to bring out the transmission engineering properties as opposed to the effects of the physical environment. An anonymous referee pointed out, “Actual construction costs of new lines exhibit significant variability, limiting the ability to utilize prospective estimates of costs in a formal optimization approach”.\(^{23}\) In the remainder of the paper we will thus focus on the reactance, however the results also translate to a more general approach regarding the lines impedance (including reactance and resistance)
Table 1: Scenario Overview for Cost Function Calculation

<table>
<thead>
<tr>
<th></th>
<th>Fixed line reactance</th>
<th>Variable line reactance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Starting line reactances</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Line extension functional parameters</td>
<td>( a_{ij} = b_{ij} = 1 )</td>
<td>( a_{ij} = b_{ij} = 1 )</td>
</tr>
<tr>
<td>Starting capacity values [MW]</td>
<td>( k_i = 0 )</td>
<td>( k_i = 2 )</td>
</tr>
</tbody>
</table>

Three node network

| FTR range [MW] | FTR 1 to 3: 1 to 5 | FTR 2 to 3: 1 to 10 |

Six node network

| FTR range [MW] | FTR 1 to 6: 1 to 5 | FTR 5 to 6: 1 to 10 |

Source: Own assumptions.

4.1 Fixed Line Reactances

In the fixed-reactance case, the loop flows are predetermined as we assume that the capacity extension does not impact the underlying line reactance/PTDF matrix. Consequently the power flow pattern cannot be altered in any sense and is defined purely by the net input \( n_i \) at nodes. The resulting cost function will represent a network with line capacities that fully resemble the power-flow on each line.

4.1.1 Extending one FTR

In the three-node system line 1 (between nodes 1 and 2) is subject to power-flows in opposite directions (counter flows) depending on the value of the two FTRs. Given a fixed level of one FTR (1→3, fixed at 2.5 MW), an increase in the second FTR (2→3) will first lead to a decrease in the flow on line 1 towards zero until both FTRs have the same value. Afterwards, the flow will again increase, although in the opposite direction. The resulting capacity cost for increasing the FTR value will show a “kink” at the level of the fixed FTR which in our example is equal to 2.5 (Figure 3, left side). In the linear and logarithmic cases the kink can clearly be distinguished. In the quadratic case the slope of the line extension function around zero is almost horizontal with gradual changes. Thus, the kink does not occur. In the lumpy case the needed capacity on line 1 is lowest when both injections cancel each other out. Any divergence from that state, no matter how small, will make it necessary to install the next integer capacity level.

24. The “kink” is similar to the one observed in a function of the absolute of a number as the capacity of a line always has to be positive no matter if the flow on a line are positive or negative: \( k_i = |pf_i| \).
which results in a sharp decrease and increase of the cost function around that point which can be observed at a level of 2.5 MW (Figure 3, left side).

In the six-node case the number of loop-flowed lines is extended to five (lines 1, 2, 5, 6, and 7). However, lines 2, 5, and 6 will cancel out their flows at the same FTR level because of network symmetry; thus, only three counter-flow kinks are obtained. Furthermore, the counter-flow on line 1 will only be observed if the FTR from node 1 to node 6 is low compared to the FTR from node 5 to 6 due to the small power-flow share caused by the second FTR on line 1. For the 2.5 MW case we only observe two kinks (Figure 3, right side), one at 1.25 MW (canceling out the flows on line 7) and the second at 5 MW (canceling out the flows on lines 2, 5, and 6). The last kink occurs at a FTR value of 20 MW, which is outside the observation range. In particular, the lumpy-cost curve shows the impact of several interacting loop flows: shortly after the first kink resulting from the counter-flow on line 7 at 1.25 MW, the cost function first increases and then decreases. This negative marginal cost range is caused by the reduced power-flow on line 1. Whereas the capacity requirement on line 1 decreases due to a decrease of the power flow on line 1 leading to a one unit lower capacity level the remaining capacities on other lines are still sufficient; consequently the overall installed capacity decreases albeit the transmitted energy increases. This effect is partly due to the integer nature of this approach which in contrast to the other three cost function cases results in “unused” capacity as a perfect matching of capacity and line flow is not possible as the former is continuous whereas the latter is integer.

4.1.2 Extending two FTRs (global cost function case)

If both FTRs are varied the resulting cost function behavior will still represent the counter-flow conditions. The presented figures show the normalized
extension costs and provide corresponding contour lines at the bottom. In the three-node case, we observe the kink of line 1 moving gradually with the increasing FTRs, which is best visualized in the logarithmic case (Figure 4, left side). The same holds true for the linear extension case, whereas the quadratic case again shows no signs of kinks. The lumpy investment case shows a more varied structure (Figure 4, right side). This is due to the combined extension of both FTRs: as the flows split up 2:1 respectively at specific extension steps, it becomes necessary to extend two or even all three lines in the system. Specifically, we see several areas with negative marginal costs. Thus, the global cost function is a combination of counter-flow based cost reductions on line 1, and possible capacity steps of 1, 2, or 3 MW depending on the net injection.

The same outcome holds true in the six-node case. The resulting global cost function shows three kinked lines (which correspond to the previous three kinked points), which again are best visualized in the logarithmic case (Figure 5, left side). The kink on the left side is associated with negative marginal costs and represents the counter-flow on line 1 which requires a low level of the FTR from 1 to 6, and a high level of the FTR from 5 to 6. The kink in the middle represents the counter-flows on lines 2, 5, and 6, and the right kink represents the flow on line 7. The lumpy investment case is again highly fragmented and shows negative marginal costs (Figure 5, right side) due to the interaction of counter-flows and capacity steps.

26. Linear and quadratic cases are presented in Figures 11–14 in the Appendix.
27. For instance, keeping the FTR from 1 to 3 fixed to 0, if the FTR from 2 to 3 is extended from 3 to 3.1 the line capacities must all be extended by 1 MW (line 1 and 2 have 1 MW, line 3 has 2 MW). This allows the increased power-flow pattern and causes a step of 3 MW.
4.2 Variable line reactances

In reality, there are limitations to extending a line capacity without altering its technical power-flow characteristics. Normally, a capacity extension is linked to a change in the reactance of the line. Therefore, in our second scenario capacity extensions are coupled with line reactances via the law of parallel circuits. Thus if a line’s capacity is doubled the line’s reactance is halved (and the PTDF matrix changes as described in section 3.1). The combined loop-flow nature of power-flows and the change in network characteristics due to capacity extensions makes a prediction of the possible outcomes more complicated.

For the following cases we assume a starting network capacity (2 MW per line). As only capacity additions beyond this starting level will result in extension costs we have zero costs as long as the initial capacities are sufficient for the desired transport volume. Therefore, the obtained results are only representing the incremental extension costs and may not represent the global cost optimum given a greenfield situation.

4.2.1 Extending one FTR

Whereas in the fixed-reactance scenario it was clear beforehand that, due to counter-flows, the power-flow on one line will fall to zero for a specific FTR combination, this may not be true in the current scenario since the canceling-out point can change with the alteration of network characteristics. This can clearly be seen in the three- and six-node cases. The current cost functions lead to no general conclusions regarding the status of counter-flows and of potentially negative marginal costs within the system (Figure 6), because line 1 is not extended for the three-node case. Increasing the capacity of line 3 will lead to a larger
The inconsistencies of the logarithmic cost function around 5 MW in the six-node case are due to the solving process utilizing Baron as GAMS solver. Using Conopt and Coinipot as solvers produces a higher cost function within that range, hinting at problems in obtaining the lowest local optimum.
Figure 7: Global Cost function, Three-node Network, Variable Reactances

Logarithmic extension costs

Lumpy extension costs

Note: The lines at the diagram bottoms indicate the contours of the cost functions.
Source: Own calculation.

4.2.2 Extending two FTRs (global cost function case)

If both FTRs are increased simultaneously, the resulting cost function in the three-node case shows cost decreases in a specific FTR range (Figure 7). These results are obtained in all extension cases including the quadratic function (see Figures 11–14 in the Appendix). However, this range has no resemblance to the kink observed in the fixed reactance case which was solely attributable to the unavoidable counter-flow on line 1. In this case the decreasing cost range is attributed to the absence of a counter-flow in the very first case (FTR from 1 to 3 is “0”). The power-flow within the system therefore is only defined by the FTR from 2 to 3 and consequently the flow on line 1 is higher than in all other cases, making an extension of this line necessary (or a much larger extension of line 3). Increasing the FTR from 1 to 3 produces a counter-flow on line 1 and leads to better utilization of the existing capacity. This effect makes the overall extension less costly (negative marginal costs both in the left and the right figure).

In the six-node case this dominant effect of a single line is canceled out by the increased number of loop-flowed lines. Within the observation range the non-lumpy cost functions show a continuous behavior with monotonically increasing global costs for increasing FTR values (Figure 8). The lumpy case shows an increasing cost pattern without a large fragmentation within the observation range.

4.3 Discussion

The potential problems of transmission cost functions alluded to earlier derive from loop-flows that may produce decreasing or even negative marginal costs and discontinuities. Theoretically these problems can be solved with free
Figure 8: Global Cost Function, Six-node Network, Variable Reactances

Note: The lines at the diagram bottoms indicate the contours of the cost functions. 
Source: Own calculation.

disposal, but electricity cannot be freely disposed of. Table 2 presents a summary that compares the cases addressed in Sections 4.1 and 4.2. We can draw some conclusions regarding the relationship of kinks, negative slopes and loop-flows for the non-lumpy cost functions. First, we observe that smoothness is gained with variable reactances in the three-node case. Whereas in the fixed-reactance scenario it was clear that for specific FTR combinations the power-flow on one line will fall to zero and cause kinks in both the 1-FTR and global-cost function cases, this is not valid in the variable-reactance scenario because the canceling-out points can change with the alteration of network characteristics. Additionally, for the global-cost function case the counter-flow structure may imply decreasing ranges of the cost function.

The analysis for the six-node case is richer because of its complex network topology and loop-flow structure. However, in the global-cost function case the same conclusion prevails. Continuity in the cost function is gained when reactances are allowed to vary. Non-lumpy cost functions then show a smooth behavior but with increasing global costs for increased FTR values. The above results apply to the lumpy extension case where by definition, network expansion is carried out within a discontinuous environment. The obvious difference is that the gains from considering variable reactances will imply an increasing stepwise

29. A downward-sloping total cost curve for an FTR (i.e., with negative marginal costs) means that a larger FTR is provided at a lower cost. The question is if the unused part of the FTRs can be thrown away. It could be argued that the additional amount of FTRs can only be provided by actually injecting and taking out the required additional electricity. This would incur an additional cost of generation, which would contradict free disposal. It would nevertheless be theoretically possible that (through non-use of part of the generated electricity) the total costs of electricity generation plus transmission exhibits no negative marginal costs even though marginal costs of transmission are negative.
### Table 2: Overview of Results

<table>
<thead>
<tr>
<th>Function</th>
<th>Fixed Reactance</th>
<th>Variable Reactance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 FTR Fixed</td>
<td>2 FTRs</td>
<td>1 FTR Fixed</td>
</tr>
<tr>
<td>Three nodes</td>
<td>• Resulting capacity cost for increasing an FTR value will show a “kink” at the level of the fixed FTR.</td>
<td>• The kink of lines moves gradually with increasing FTRs.</td>
</tr>
<tr>
<td>Six nodes</td>
<td>• In the lumpy extension case, the kink is represented by a jump in the step function.</td>
<td>• The lumpy investment case is a combination of cost reductions based on counter-flows and capacity steps depending on net injections.</td>
</tr>
<tr>
<td></td>
<td>• In the quadratic case the slope of the line extension close to the origin is almost horizontal, thus the loop-flow kink does not occur.</td>
<td>• The quadratic case shows no signs of kinks.</td>
</tr>
<tr>
<td></td>
<td>• Lines 2, 5, and 6 will cancel out their flows at the same FTR level due to the network symmetry, and thus three counter-flow kinks are obtained.</td>
<td>• Resulting global cost function shows three kinks.</td>
</tr>
</tbody>
</table>
Long-run Cost Functions for Electricity Transmission / 151

Continuity of cost functions is a concrete advantage of a transmission cost analysis based on FTRs instead of power-flows.\textsuperscript{30} This property is crucial for the application of price-cap incentive mechanisms to real-world expansion projects. Especially when we assume increasing returns to scale, the resulting capacity extension pattern includes sudden shifts in the extended lines which in turn cause a significant alteration in power-flows. When we define the transmission output as line power-flows, the shifts will cause jumps in the resulting cost functions. Whereas the redefinition via an FTR approach only takes into account the overall extension costs and thus avoids line specific discontinuities.

A further issue that would generalize this analysis considers a more realistic scenario of electricity networks. When cost minimization occurs over the optimal design of the network (i.e., the location and number of links and nodes denoted by the PTDF matrix), the network topology is affected accordingly. In this case, the structure of the PTDF matrix becomes a function of the chosen line capacities, and a more complicated cost minimization problem results.\textsuperscript{31} The problem also leads to new goods (FTRs) for new nodes, and these new goods change the costs for all of the old goods. Hence, all FTRs are affected if free nodes are added or removed.

One way to first address this problem is to consider an incremental change in network architecture. The method would calculate the cost function for the changed network, and then compare it to the cost function of the original network. If the new cost function lies everywhere below (above) the original cost function, the new topology dominates (is dominated by) the original one. Most likely, as suggested by our previous simulations, one will dominate the other only over part of its range.

Another solution is to evaluate ways to minimize costs for the network topology alternatives. If, for simplicity, we assume that the nodes are given by the location of power stations and load centers, one has to choose between different line configurations. The number of configurations will grow quickly with the number of nodes. But after identifying all cost functions for all topologies, the long-term cost function will be the minimum cost locus (the lower envelope convex hull).

\textsuperscript{30} An anonymous referee suggested that it may be the case that considering security constraints makes the optimal cost function even more well-behaved. Considering such constraints will be the topic of further research.

\textsuperscript{31} We conjecture, however, that in the cases with two generation nodes and one consumption node dealt with in the current paper the optimal network architecture underestimates the real world complexity. Our results might become more differentiated with respect to decreasing cost elements and the overall cost function shape when many generation nodes and many consumption nodes are considered. Results may also be different if one adds reliability constraints, such as an (n-1) constraint. We plan to address these issues in future work.
5. APPLICATION TO A REGULATORY MECHANISM FOR TRANSMISSION EXPANSION

The HRV (2010) model combines the merchant and PBR transmission approaches in an environment of price-taking generators and loads. Its crucial aspect is the redefinition of the transmission output in terms of incremental FTRs in order to apply the basic price-cap mechanism in Vogelsang (2001) to meshed networks within a power-flow model. The Transco intertemporally maximizes profits subject to a cap on its two-part tariff, but the variable fee is now the price of the FTR output based on nodal prices. The rebalancing between the variable and fixed charges promotes the efficient expansion of the network. Under the HRV incentive mechanism, investments will continue through time until they converge to an optimal (Ramsey-price) level. However, this only holds under the assumptions that transmission’s demand functions are differentiable and downward-sloped and that the marginal cost curves cut demands only once. However, our observations above (in Figure 3 right, Figure 4 left and right, Figure 5 right and Figure 7 left and right) exhibit areas of negative marginal costs (where the total cost function either jumps down or has a negative slope). This is something that never happens with “normal” cost functions and could potentially violate the assumptions necessary for the working and convergence of such mechanisms. The local presence of such anomalies, however, does not necessarily mean that the mechanism would not converge or would not converge to the optimum. The firm may end up at a local, not global optimum, though. It appears that these concerns might not always be relevant, as the following simulation results by Rosellón and Weigt (2011) and Rosellón, Mislíková and Zenón (2011) illustrate.

Applying a simple three-node setting as presented in Figure 2, these two last studies show that the regulatory mechanism grants convergence towards the welfare optimal solution over time. The obtained results are robust to changes in the underlying network, demand, and generation assumptions. Extending the model approach to represent the North-West European electricity market with the Netherlands, Belgium, France, and Germany the results obtained by Rosellón and Weigt (2011) show a convergence of price levels within the region due to the extensions carried out by a regulated Transco (Figure 9) as well as a convergence of the regulatory approach towards the welfare benchmark (Table 3). A similar result is obtained for the Pennsylvania, New Jersey, and Maryland (PJM) area (Figure 10, Table 4). The network representation with 15 nodes and 28 lines in Rosellón and Weigt (2011) and with 17 nodes and 31 lines in Rosellón, Mislíková and Zenón (2011) exceed the range of the networks analyzed in this paper and consist of several injection points, counter flow situations, and congestion problems. Although, the actual transmission costs in these exercises have not been derived, the results suggest that the conclusions drawn from the cost function analysis could also hold for more complex network topologies.

32. In both the North-West Europe and PJM cases, convergence is achieved in terms of consumer surplus, producer surplus, total welfare, and grid capacity (see Tables 3 and 4).
Figure 9: Price Development in the European Model


Table 3: Comparison of the Regulatory Approach with Welfare Maximization for the Benelux Area

<table>
<thead>
<tr>
<th></th>
<th>No grid extension</th>
<th>Regulatory Approach</th>
<th>Welfare Maximization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer rent [Mio€/h]</td>
<td>10.37</td>
<td>10.31</td>
<td>10.30</td>
</tr>
<tr>
<td>Producer rent [Mio€/h]</td>
<td>0.65</td>
<td>0.99</td>
<td>1.02</td>
</tr>
<tr>
<td>Congestion rent [T€/h]</td>
<td>107.8</td>
<td>20.20</td>
<td>7.13</td>
</tr>
<tr>
<td>Total welfare [Mio€/h]</td>
<td>11.13</td>
<td>11.32</td>
<td>11.33</td>
</tr>
<tr>
<td>Total extension sum [Mio€]</td>
<td>—</td>
<td>285.27</td>
<td>305.26</td>
</tr>
<tr>
<td>Total grid capacity [GW]</td>
<td>33.4</td>
<td>60.9</td>
<td>62.64</td>
</tr>
<tr>
<td>Average price [€/MWh]</td>
<td>28.4</td>
<td>18.5</td>
<td>18.1</td>
</tr>
</tbody>
</table>

* Excluding auxiliary lines.

* Excluding auxiliary nodes.

Source: Based on Rosellon and Weigt (2011).
Figure 10: Price Development for the PJM Region


Table 4: Comparison of the Regulatory and Benevolent ISO approach for PJM Region

<table>
<thead>
<tr>
<th></th>
<th>No grid extension</th>
<th>Regulatory Approach</th>
<th>Welfare Maximization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Rent (MioUSD/h)</td>
<td>6.53</td>
<td>6.63</td>
<td>6.67</td>
</tr>
<tr>
<td>Producer Rent (MioUSD/h)</td>
<td>0.36</td>
<td>0.59</td>
<td>0.64</td>
</tr>
<tr>
<td>Congestion Rent (MioUSD/h)</td>
<td>0.067</td>
<td>0.01</td>
<td>0.006</td>
</tr>
<tr>
<td>Total Welfare (MioUSD/h)</td>
<td>6.95</td>
<td>7.23</td>
<td>7.32</td>
</tr>
<tr>
<td>Total Grid Capacity (GW)</td>
<td>35.8</td>
<td>50.83</td>
<td>52.83</td>
</tr>
<tr>
<td>Average Price (USD/MWh)</td>
<td>53.64</td>
<td>43.11</td>
<td>42.97</td>
</tr>
</tbody>
</table>

6. CONCLUSION

We have analyzed the cost functions of electricity transmission when the transmission output is redefined in terms of FTR point-to-point transactions. We were motivated to do so because smooth, well-behaved cost functions may not hold in a meshed network with loop-flows. Likewise, a conventional definition of the electricity transmission output is not possible since the physical flow through loop-flowed meshed transmission networks obey Kirchhoff’s laws. We explicitly tried to provide more evidence for the intuition suggested in the literature that ill-behaved non-continuous transmission cost functions are mainly due to capacity changes that modify network topology. We therefore focused on fixed topology networks with two cases, one where line capacities could change but not the lines’ reactances, and the other where changes in line reactances linking capacity extensions and power-flow distribution are allowed.

Our simulations in general suggest that FTR-based cost functions remain piecewise continuous—as well as piecewise differentiable—over the entire FTR range. Regions with negative marginal costs could cause concern. However, the regions seem to shrink, as one moves to more realistic (multi-node) networks and as one includes changes in reactances in the model. This then provides support for applications of mechanisms that use FTRs to promote network expansion. More specifically, our results showed that compared with the fixed-network case the introduction of variable line reactances significantly changes the possible outcomes. In particular, the introduction of a link between capacity and reactance appears to reduce the impact of loop-flows in terms of significant kinks. Therefore, one general result is that smoothness of non-lumpy cost functions is gained with variable reactances. In a lumpy environment, this result translates to variable reactances, implying increasing stepwise functions.

Our approach provides a first methodological step towards empirical cost estimations for transmission networks using simulations and towards empirical estimates of actual transmission network costs. This should also help to improve future regulation of transmission networks compared to practices nowadays. The current status quo is primarily based on cost-of-service regulation, which in general lacks a normative academic framework. Regulations across international electricity systems rarely make reference to cost-minimization incentives, while Transcos and RTOs engage in transmission expansion projects without a theory-based planning scheme.

Overall, our results also reveal the difficulties that electricity networks present when applying standard approaches. Even for a simple extension, loop-flows can lead to a mathematically complex global cost function. This in turn makes the estimation of revenue and profits more complex in a general setting. Additionally, the link between capacity extension and line reactances (and thus flow patterns) produces results that are highly sensitive to the grid structure. For modeling purposes, the logarithmic and lumpy behaviors produce high degrees of nonlinearities with non-smoothness, and require further calculations and solver
capabilities, the quadratic functions show a generally continuous behavior, and the linear extension functions fall somewhere between. Most suitable for modeling, therefore, is combining the latter with the piecewise, linear nature of the resulting global costs function, making it possible to derive global optima.

Nevertheless, an analysis of investment incentives in electricity transmission relies on numerical analysis to capture the physical nature of the network, so that conclusions remain feasible within a range of systems and cases. One challenge for future research is the examination of the external influences (e.g., geographical conditions or different states of nature) which might cause dysfunctional behaviors with sudden slope changes.

Further research steps would also include the derivation of cost functions based on expansion FTRs such as suggested in the HRV (2010) mechanism. In each period, the demand for such incremental FTRs would determine marginal costs for increased transmission capacity which would in turn affect demand for FTRs in the next period. This would provide an intertemporal and path-dependent\[sup]33\] cost function for each specific expansion project. It also would allow us to deal with growth in demand over time. Further complexities in the HRV (2010) model would in turn imply complexities in the associated cost-function analysis. Some more complex settings include: market power in energy FTR markets, a cost function unknown to the regulator, different states of nature, and a regulatory mechanism that combines low-powered incentive schemes for the long term and high-powered incentive schemes for the short term. Other issues would further address the impacts on the cost function behavior of lack of fiscal transfers (so that the fixed-fee allocation among consumers might have distortive effects) as well as capacity reserves. Another interesting expansion would be to include reliability in simulations of the security-constrained dispatch and in the calculation of nodal price differences by the ISO. This would build redundancy into the model in order to make sure that each load gets served if either a line or a power station can fail. This additional model feature could further allow us to deal with short-term demand fluctuations.

The ultimate research challenge, however, is to identify the changes in network topology and the alternatives that minimize costs throughout the system.\[sup]34\] Recognizing that the number of configurations increases significantly with the number of nodes, nonetheless an exhaustive analysis of the properties of transmission cost functions should suggest future research to pursue.

Once these challenges have been resolved, the emphasis should be on empirical assessments of line costs in order to combine our analytical approach

\[sup]33\] We in fact already have some path dependence in our simulations of the case with variable impedances (section 4.2).

\[sup]34\] Such an analysis could be further carried out considering cheap and dear nodes where there is a trade-off between “lumpy investment in transmission and no investment in generation” vis a vis “no investment in transmission and investments in generation.” Thus, one could look at minimizing total electricity costs at the consumption nodes including both transmission and generation costs.
with actual (engineering) data so that actual cost functions can be estimated in ways pioneered by Baldick and Kahn (1993) or for telecommunications networks by Gasmi et al. (2002). As an anonymous referee has pointed out, these line costs are extremely variable. “Facing this difficulty, and perhaps as well for other reasons, regulators have apparently made little effort to make reference to cost minimization. Transmission companies have presumably followed suit, particularly in the US where cost-of-service regulation persists for transmission even in restructured markets.” We may, however, have reached a point in time where these arguments no longer hold. Transmission costing has become more important because (a) the new independent Transcos can no longer hide transmission costs in overall costs and have to price their services independently, (b) siting of unconventional generation (wind, solar) requires new lines over sometimes large distances. Variability of costs is a big issue in telecommunications network costing as well. However, in that industry cost analysts have been able to link line costs to geography (soil, presence of bodies of water, mountains, etc.) and population density to come up with computerized relationships. We are far away from that in electricity transmission but our approach could be a first step.

REFERENCES


Long-run Cost Functions for Electricity Transmission / 159


**APPENDIX**

**Figure 11: Global Cost Function, Three-node Network, Fixed Reactances**

<table>
<thead>
<tr>
<th>Linear extension costs</th>
<th>Quadratic extension costs</th>
</tr>
</thead>
</table>

Note: The lines at the diagram bottoms indicate the contours of the cost functions.

Source: Own calculation.

**Figure 12: Global Cost Function, Six-node Network, Fixed Reactances**

<table>
<thead>
<tr>
<th>Linear extension costs</th>
<th>Quadratic extension costs</th>
</tr>
</thead>
</table>

Note: The lines at the diagram bottoms indicate the contours of the cost functions.

Source: Own calculation.
Figure 13: Global Cost Function, Three-node Network, Variable Reactances

Linear extension costs

Quadratic extension costs

Note: The lines at the diagram bottoms indicate the contours of the cost functions.
Source: Own calculation.

Figure 14: Global Cost Function, Six-node Network, Fixed Reactances

Linear extension costs

Quadratic extension costs

Note: The lines at the diagram bottoms indicate the contours of the cost functions.
Source: Own calculation.